The Theory of Relativity.[1])

From

A. EINSTEIN in Prague.

The sole cornerstone upon which the theory known as the "theory of relativity" rests is the so-called *principle* of relativity. Thus, I first want to clarify what this principle of relativity means. Picture two physicists, each equipped with every imaginable physics apparatus and each with a laboratory. One laboratory sits somewhere in an open field, while the second rides in a railway car, moving at constant speed in an unchanging direction. The principle of relativity says the following: If these two physicists — one unmoving in a field and the other riding in a railway car that travels evenly and without discernable shaking — apply all of their devices, they discover precisely the same laws of nature. We can say that a bit more abstractly: According to the principle of relativity, the laws of nature are independent of the translational movement of the reference system.

Let us consider the role this relativity principle plays in classical mechanics. Classical mechanics is based primarily on the Galilean principle, in which a body not subject to the influence of other bodies remains in straight, uniform motion. If this sentence applies to one of the laboratories mentioned above, it also applies to the second. We can

---

[1]) Lecture given at the Natural Research Society meeting in Zurich on January 16, 1911.

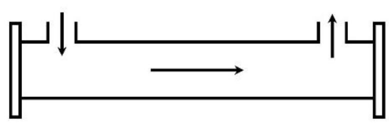
gather this directly from intuition, but we can also derive it from the equations of Newtonian mechanics if we transform those equations into a reference system that moves uniformly relative to the original.

(Incidentally, you may have noticed that I always talk about laboratories for measurements. In mathematical physics, things are not usually related to a specific laboratory but rather to coordinate systems. What is essential in both cases is the following: If we say anything about the location of a point, we always state it in terms of the colocation of this point with a point of a certain physical system. If, for example, I take myself as this material point and say, "I am at this point in this hall," then I have either brought myself to coincide in spatial relation with a certain point in this hall or have expressed an existing colocation. I do this in mathematical physics by using three numbers, the so-called coordinates, to describe the location of the point in terms of where it coincides with points of a rigid system, which is called the coordinate system.)

The principle of relativity that I just described is fully general. If you had asked a physicist from the 18th century or the first half of the 19th century whether they had any doubts about this principle, they would have answered the question emphatically in the negative. They had no reason to doubt it since people at the time firmly believed that the laws of classical mechanics were the source of all natural events. ¶ I now want to explain how experimental results have led physicists to propose physical theories that contradict this principle. To do this, we must look briefly at the gradual development of optics and electrodynamics over the last few decades from the perspective of the principle of relativity.

Like sound waves, light shows interference and diffraction, so many have felt moved to view light as a wave movement or, more generally, as a periodically changing state of a medium. This medium was called the ether. Until recently, the existence of such a medium seemed certain to physicists. Although the theory outlined below is incompatible with the ether hypothesis, we want to hold on for now to the ether concept to help us see how ideas about this medium developed and what kinds of questions arose from introducing this physical theory that presupposes the ether. ¶ We have already said that physicists imagined light to consist of vibrations of a medium, that is, that this medium provides propagation of the vibrations we call light and heat. As long as one was concerned exclusively with the optical phenomena of bodies at rest, one had no reason to ask about movements of this medium other than those that enable light and heat. Physicists assumed both the medium and the material bodies within it to be in a simple state of rest, with the only movements involved being the oscillations that comprise light and heat.

However, when we began looking at optical phenomena in moving bodies and the related issue of the electromagnetic properties of moving bodies, the question of how the light ether behaves when using light to observe bodies in motion could no longer be avoided. Does the light ether move with such bodies at every location, or is that not the case? The simplest assumption is that the light ether moves with the matter at all locations. A second simple possibility is that the light ether takes no part whatsoever in the movements of matter. In this second case, interactions between matter and the light ether are possible. The main characteristic of such interactions would be that the ether in a region of space could, to some degree, move independently of the matter in that same region. ¶ We now want to look at how people tried to get an answer to such questions. The first result came from an especially important experiment by French physicist Fizeau. This experiment owes its existence to the question presented by this figure:



The tube sketched above has glass plates sealing its back and front ends. Input and output ports near both ends enable a liquid to flow through the tube along its long axis. The question, then, is whether and how the speed at which the liquid flows through the tube influences the speed of propagation of a light ray passing through the long tube axis. ¶ If it is true that the light ether moves with the matter that flows through the tube, we have the following situation: If we assume that the propagation of light in still water occurs at the speed V, let that be its speed relative to the water, and let v be the speed of the water relative to the tube. We must then say that the speed of the light relative to the water is always the same if the light ether adheres to the water, regardless of whether the water is or is not moving. It is, therefore, expected that the speed of light propagation relative to the tube for a moving liquid should be v greater than for a liquid at rest. ¶ In Fizeau's experiment, he split a single beam to produce two interference-capable light beams, one of which he passed through the tube as described and the other of which stayed outside the tube. From the influence of the known speed of movement of the liquid on the position of the interference fringes when these beams recombined at the far end, Fizeau could calculate how large an influence the movement of the water at speed v had on the speed of light propagation relative to the stationary tube. ¶ What Fizeau found was that the speed of light relative to the tube does not increase by the speed v as a result of the movement of the liquid, but only by a fraction of this amount: $v\left(1-\frac{1}{n^2}\right)$, where n is the refractive index of the liquid. If the refractive index of the liquid is almost $= 1$, that is, if the light propagates in the liquid almost as quickly as in empty space, then the movement of the liquid has almost no influence. From this result, one must conclude that the idea that light always propagates at the same speed relative to water is incompatible with experience.

The next simplest hypothesis was that the light ether entirely ignores the movements of matter. However, with this hypothesis, it is not easy to deduce how the movement of matter influences optical phenomena. Nonetheless, H. A. Lorentz managed in the mid-1890s to put forward a theory that assumes the light ether to be completely immobile. His theory fully and correctly reproduces almost all known phenomena in the optics and electrodynamics of moving bodies, including the Fizeau experiment just mentioned. I also wish to be clear that no one managed to put forward a theory with simple and clear assumptions fundamentally different from Lorentz's theory yet capable of achieving the same objectives. Therefore, without further developments, everyone had to accept the theory of a resting light ether as the only one reconcilable with the totality of experimental results.

It is now time to consider the resting ether theory from the perspective of the principle of relativity. If we define acceleration-free systems as ones whose material points are not subject to external forces and thus move uniformly, then the principle of relativity says this: The laws of nature are the same for all acceleration-free systems. On the other hand, among all possible acceleration-free motion systems, Lorentz's basic hypothesis of the resting light ether unavoidably distinguishes one acceleration-free motion state: systems at rest relative to this light medium. ¶ So even though one can never say there is an absolute movement in the philosophical sense since it is only possible to conceive of motion as relative changes in the position of bodies, one nonetheless can define an absolute state of movement in the physical sense that we do have preferred a state of motion: that of bodies at rest relative to the ether. Thus, we can describe a body as, in a sense, absolutely resting if it is at rest relative to the wave medium of light. Reference systems at rest relative to the aether thus are distinguished from all other acceleration-free reference systems. In this sense, Lorentz's concept of a resting light ether does not do justice to the principle of relativity. ¶ Basing our view of light on the concept of an ether at rest leads to the following general consideration: Assume a reference system k to be at rest relative to the light ether. Let another reference system k' move uniformly relative to the light ether. The movement of k' relative to the ether should then influence how the laws of nature apply relative to k'. Physicists thus expected that measuring the laws of nature relative to k' should give different results from measuring them relative to k, due to the motion of the light ether relative to k'. ¶ It must also be said that due to the motion of the Earth around the sun, laboratories residing on Earth cannot possibly be at rest relative to this light medium throughout the year. Thus, any laboratory must, at various times over one year, play the role of a reference system k' that is in motion relative to the ether. Since the physical space we encounter on Earth should behave differently in different directions due to Earth's movement relative to the ether, the implication was that there should exist phenomena in which this relative movement of the ether affects experiments in our laboratories. But it has not been possible to prove anything of the sort in even a single case.

Everyone was now in an unpleasant position regarding the ether. Fizeau's experiment showed that the ether does not move with the matter, thus implying that the light medium must move relative to matter. On the other hand, all attempts to see this relative movement produced a negative result. Thus, these two outcomes seemed to contradict each other, and it was incredibly painful for physicists that they could not eliminate this unpleasant dichotomy. There was a growing worry that it might prove impossible to reconcile the principle of relativity — to which no exception could be found, despite much searching — with Lorentz's theory. ¶ However, before we go deeper into this, we must point out the most important point that follows from Lorentz's theory of a resting light ether: What does it mean physically to say that a resting light ether exists? The most important implication of the resting light ether hypothesis is this: There exists a reference system, which Lorentz's theory calls the "system at rest relative to the ether," from which every measurement of how a light ray propagates in a vacuum should give a single universal speed c. [Experimentation, however, shows that this nominally special case of a single universal light speed applies regardless of whether the light-emitting body is at rest or in motion.] We can call this broader statement the principle of *constancy of the speed of light*. This phrase allows us to restate our earlier dilemma: Can we reconcile the *principle of relativity*, which experiments seem to indicate is fulfilled without exception, with this new principle of the *constancy of the speed of light*?

There is an obvious scenario that speaks against such reconciliation. If every light ray measured from reference system k propagates with speed c, and a second reference system k' is in constant motion relative to k, it should be impossible for all of those light rays to move at speed c when measured from reference system k'. For example, if k' moves with speed v (relative to k) in the same direction of propagation as a particular light ray, then from the rules of physics with which we are familiar, the propagation speed of the light ray when measured from k' should equal $c - v$. Thus, the laws of light propagation observed from k' would differ from the laws of light propagation relative to k. This disparity would be a violation of the principle of relativity. ¶ This is a terrible dilemma, but it turns out that nature is completely innocent in its cause. Instead, the dilemma arises because we relied on tacit and arbitrary assumptions in our considerations, including the light ray scenario I just described using k and k'. We must, therefore, identify and abandon these assumptions before we can arrive at a consistent and simple understanding of things.

What are these arbitrary assumptions in the foundations of our physical thinking, and how do we disentangle them? The first and most important arbitrary assumption concerns our concept of time, so I must attempt to explain the nature of this assumption and why it is arbitrary. However, to do this well, I must first deal with space since this approach gives me a foundation for dealing with time in a more parallel fashion. ¶ If we want to express the position of a point in space, such as the position of a point relative to a coordinate system k, we specify its rectangular coordinates x, y, z. The method for finding these coordinates is as follows: You construct perpendiculars from the point to the three coordinate planes according to known rules, then count how many times a given standard length unit fits onto those perpendiculars. The results of these counts are the coordinates. A set of spatial coordinates thus is the result of well-defined manipulations. Such spatial coordinates thus have a highly specific physical meaning, without which it becomes impossible to verify whether or not a given point has the specified coordinates.

But what about time in this relationship? We see that on that point, we are not doing so well. Until recently, people have always been content to assume time is the independent variable when specifying an event. However, such a definition is insufficient for measuring the time of an actual event. For that, we must instead define time in a fashion that makes physical measurements of its value possible. ¶ We imagine placing a clock, such as a balance-wheel clock, at the starting point of a coordinate system k. With this addition, measuring the time of events occurring at or very near the starting point becomes possible. However, notice that the time of events at points far away from the k starting point cannot be evaluated directly using this clock. If an observer standing near the clock at the starting point of k notes the time at which he sees a distant event occur, then this time is not the time of the event itself but a time that includes the speed of propagation of the light ray from the event to the clock, and so is greater than the time of the event. ¶ However, if we know how long it takes light to propagate from the distant event to the clock at the starting point, as measured from system k, the time of the distant event can be determined using the clock at the starting point. The catch is that obtaining a number for how long it took light to travel from the event to the origin requires knowing when the light left the distant event, which subtly presumes we have already solved the problem of how to measure time at distant points. ¶ To understand why, notice that measuring the speed of light propagation from point A to point B requires measuring both the distance between A and B and the time delay from the emission of the light at A and its arrival at B. However, measuring the delay requires knowing the times at different locations, that is, at both

A and B. Thus, measuring the delay is possible only if we already have the time definition we seek. ¶ On the other hand, since we now know that the lack of a preexisting universal time definition makes it fundamentally impossible to measure any speed, particularly the speed of light, without first applying an arbitrary determination, we are entitled to make just such an arbitrary stipulation regarding how light propagates. We stipulate only this: The speed of light propagation in a vacuum from A to B is the same as from B to A. ¶ Thanks to this stipulation, we can now add more clocks of the same type at various locations, all at rest relative to the system k. For example, we can place similar clocks at A and B and stipulate that if A at time t (as measured on the clock at A) sends a light beam to B, which arrives at B at time $t + a$ (as measured on the clock at B), then *by definition* a light beam sent from B towards A at time t (as measured on the clock at B) must also arrive in A at time $t + a$ (as measured on the clock at A). ¶ This is the rule according to which all clocks distributed in system k must be adjusted. By enforcing this constraint, we obtain a way of determining time that a physicist can apply practically: The time of an event is equal to the time given by whichever previously adjusted coordinate clock is closest to the event.

Since this all sounds self-evident, the question is whether this result is particularly strange. What is strange is this: To give event times a very specific meaning, one must apply the rule of setting clocks by assuming equal light propagation times in both directions between any two clocks to a large number of clocks that are all at rest relative to a very specific coordinate system k. That is, we have not obtained a definition of time *per se*, but only a definition of time relative to the coordinate system k — or, more precisely, with reference to the clocks arranged at rest relative to the coordinate system k. ¶ We can, of course, perform precisely the same operations if we have a second coordinate system k' that is in uniform motion relative to k. We can distribute another clock system over space relative to this coordinate system k', but in such a way that the clocks move with k' and are at rest relative to it. We can then adjust these clocks at rest relative to k' using the same rule given above. If we do this, we get a new definition of time with respect to the system k'.

Notice, however, that one can no longer assert *a priori* that if two events are simultaneous relative to system k that these same events are simultaneous relative to system k'. Time no longer has an absolute meaning since this theory binds it to the state of motion of its reference system. The first arbitrary assumption in our kinematics — in our physics of motion — thus was the belief that time has an absolute meaning independent of any physical reference system.

We now come to the second arbitrary assumption that affected our earlier understanding of kinematics. We speak of the dimensions of a body — for example, the length of a rod — and believe we know its exact length even if it is in motion relative to the reference system used to observe its motion. However, a quick reflection shows that these concepts are not as simple as we instinctively imagine. Suppose that a rod is moving in the direction of its long axis relative to reference system k. We now ask: How long is this rod? To make this question meaningful, we must ask: What experiments must we perform to determine the length of the rod? ¶ One approach would be to give a man carrying a ruler a hard enough push to match the direction and velocity of the rod. The push places him at rest relative to the rod and allows him to determine the length of this rod by repeatedly applying his ruler just as he would to find the length of a body at rest. He gets a specific number and can declare, with some justification, that he has measured the length of this rod.

But what if the only observers available cannot move with the rod but must remain at rest relative to reference system k? In this case, we can proceed in the following way. We again imagine the rod moving relative to k on its long axis. This time, however, we prepared ahead by distributing many clocks along the anticipated path of the rod, each with an observer attached to it. Using the experimental procedures given earlier, we adjusted the clocks using exchanges of light signals so that, as a whole, they show the definition of time that belongs uniquely to the k reference system. At some previously agreed-to time t, all these observers look to see if the front or back end of the rod is at their location. Since each observer has a clock showing k time, this is the same as finding the two k clocks co-located at the moment t with the front and back ends of the rod. Another observer in k then completes the process by finding the path distance between the two observers (or their k clocks) using the usual procedure of applying a ruler he carries.

¶ Notice that the results of either of these methods — measurement by an observer moving with the rod or by observers watching the rod pass — can rightly be defined as measuring the length of the moving rod. The difficulty is that these two procedures do not necessarily give the same result! In other words, our second arbitrary assumption was that the geometric dimensions of a body are independent of the state of motion of the reference system used to specify those dimensions.

However, abandoning these two arbitrary assumptions prevents us, at least initially, from solving the following elementary problem: Given the coordinates x, y, z, and the time t of an event observed from system k, how do we find the space-time coordinates x', y', z', t' of the same event relative to another system k' that is in known, uniform translational motion relative to k? Unfortunately, our earlier simple approach to this problem relied on the two assumptions we now recognize as arbitrary.

How should we get the kinematics back on track? The answer is self-evident: The same circumstances that previously caused us embarrassing difficulties now lead us onto a viable path. We have gained more leeway by eliminating the aforementioned arbitrary assumptions. The two seemingly incompatible principles that experimental results forced upon us — the principles of relativity and constant light speed — now lead to a very specific solution to the problem of space-time transformation. However, they also lead to results that sometimes fully contradict our everyday expectations. The mathematical considerations that lead to this are simple, but this is not the place to go into them. [1]) Since these principles arise through a completely logical process that requires no further presuppositions, we can instead go into the main consequences.

---

[1]) If x, y, z, t and x', y', z', t' are, respectively, the space and time coordinates of the two reference systems k and k', the two underlying principles require that the transformation equations must be such that from the two equations

$$x^2 + y^2 + z^2 = c^2 t^2 \text{ and}$$
$$x'^2 + y'^2 + z'^2 = c^2 t'^2$$

each result in the other. For reasons beyond this discussion, the substitution equations must be linear. A short examination shows that this constraint fully determines the transformation equations. (See, e.g., Jahrbuch der Radioaktivität und Elektronik IV. 4, p. 418 ff).

The first consequence is purely kinematic, a consequence of motion. Since we have defined coordinates and time in a specific physical way, every relationship between spatial and temporal quantities also has a specific physical meaning. One result is that if observers in coordinate system k observe a body moving uniformly past them, the moving body appears shortened in its direction of movement. This shortening is directly related to the shape of the body when it is at rest. If c is the speed of light, v is the speed of movement of the body as viewed from k, and l is any length of the body in the direction of motion while it was still at rest, imparting the velocity v to the body reduces the length l as measured by an observer at rest in k to the following lesser amount:

$$l \cdot \sqrt{1 - \frac{v^2}{c^2}} \, .$$

Thus, if the body is spherical at rest, it has the shape of an oblate ellipsoid when in motion in a certain direction. When the speed reaches the speed of light, the body collapses into a plane. However, when judged by a moving observer, the body retains its spherical shape. Notice also that for an observer moving with the body, all objects not moving appear shortened in exactly the same way in the same direction of the relative movement. This result loses much of its strangeness when one considers that this information about the shape of moving bodies has a rather complicated meaning that, as in our earlier analysis, can only be understood with the help of time determinations.

The impression that the term "shape of the moving body" has an immediately obvious meaning comes from the fact that, in everyday experience, we usually encounter only velocities that are, for all practical purposes, infinitely small compared to the speed of light.

A second purely kinematic consequence is perhaps even stranger. We imagine a clock that, when at rest relative to k, keeps the same time as the reference clocks we previously distributed and adjusted throughout k. One can prove that if we put this clock into uniform motion relative to k, it runs slower than the k clocks. More precisely, when a k observer sees the moving clock added 1 unit of time at the moment it passes by, the k clock associated with that observer shows the addition of the larger number of time units

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\,.$$

In other words, the moving clock runs slower than the clocks at rest with respect to k. ¶ To visualize how this measurement process works, imagine comparing the hand positions of the moving clock to the hand positions of each k observer's clock just as the moving clock passes. If the moving clock can travel at the speed of light — and we could move it at approximately that speed if we had enough power — then, judging from the reference clocks of k, the hands of the moving clock would advance infinitely slowly.

Things get the funniest if you consider this example: Impart a very high speed, equal almost to c, to a clock, and let it continue in uniform motion. After it has flown a long distance, give it an impulse in the opposite direction so that it returns at the same high speed to its point of origin. The result is that even though the hands of this clock have hardly moved over its entire journey, the hands of a similar clock that remained at rest at the origin have changed dramatically. ¶ It is worth adding that since we introduced this clock as a simple representative of all physical systems, whatever applies to it applies to a self-contained physical system of any nature. If, for example, we were to put a living organism in a box and send it out and back in the same fashion as the clock, we could return this organism after undergoing a flight of any desired length to its original location with as little change as desired. Meanwhile, organisms of the same type that remained at rest at the origin would long since have given way to new generations. Yet, for the organism that took this trip at approximately the speed of light, only a moment of time passed! This unexpected result is an unavoidable consequence of the principles forced upon us by experience.

I now want to say a few words about the importance of the theory of relativity for physics. This theory requires that for the mathematical expression of a natural law to remain valid at any speed, the formulas that express the law must not change form when new space-time coordinates are introduced by applying the relativity transformation equations. This constraint significantly limits the range of acceptable formula options. ¶ Also, by applying a simple transformation to laws already known for bodies at rest or moving slowly, one can derive more general laws that apply to bodies moving at any speed. The laws of motion for fast cathode rays are an example. Newton's equations do not apply to material points moving at such speeds, so physicists had to develop new relativistic equations of motion with somewhat more complicated structures. The resulting relativistic laws of cathode ray deflection agree quite satisfactorily with experience.

I should mention another physically important consequence of the theory of relativity. We saw earlier that the theory of relativity says that a moving clock runs slower than the same clock at rest. Since the speeds at which we can move a large body are infinitesimally small compared to the speed of light, it likely will remain forever impossible to verify this slowing of time through experiments with pocket watches. ¶ On the other hand, nature offers objects that have the character of clocks yet are capable of moving extremely quickly, these being atoms that emit spectral lines. We can use electric fields (channel rays) to impose velocities of several thousand kilometers per second on such atoms. According to the theory of relativity, such velocities should influence the vibration frequencies of these atoms in exactly the fashion deduced for moving clocks. Even if such experiments encounter great difficulties, we can still hope to achieve an important confirmation or refutation of the theory of relativity from them in the next few decades.

The theory also leads to the important result that the inertial mass of a body depends on its energy content, but only to an extent so small that it is completely hopeless to prove the matter directly. If the energy of a body increases by E, the inertial mass increases by $\frac{E}{c^2}$. This theorem overturns the theorem of conservation of mass or, more precisely, merges conservation of mass and conservation of energy into a single law. As strange as it may sound — and even without the theory of relativity — there are special cases where one can safely conclude from experimental results that inertial mass increases with the energy content.

Finally, I would like to say a few words about the extremely interesting mathematical treatment that the mathematician Minkowski gave the theory. Minkowski, who died far too young, noted that the transformation equations of the theory of relativity are such that they have the expression

$$x^2 + y^2 + z^2 - c^2t^2$$

as an invariant. If one introduces the imaginary variable $ct \cdot \sqrt{-1} = \tau$ as a time variable instead of time t, this invariant then takes the form

$$x^2 + y^2 + z^2 + \tau^2 .$$

Here, the space and time coordinates play the same role. The further pursuit of this formal equivalence of space and time coordinates in the theory of relativity has led to a very clear presentation of this theory, making its application much easier. In Minkowski's approach, we represent physical events in a 4-dimensional space, and the space-time relationships of the results of events appear as geometric sequences in this 4-dimensional space.