How to Win a Million-Dollar Math Prize

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A Tale of Seven Prizes

- A quarter of a century ago, in 2000, the Clay Mathematics Institute (CMI) proposed seven Millennium Prize Problems, each carrying a $1 million prize
- Grigoriy Perelman solved one of them in 2002
  - The Poincaré Conjecture
  - He refused to accept the prize!
- Of the remaining six, one stands out
  - It's not really a math problem
  - It's a physics problem that CMI hoped could be solved by math alone
- Its name is the **Yang-Mills Positive Mass Gap Challenge**

Part I. Warnings!

Warning #1: If You Win, Don't Plan on Getting the Money

- The Clay Math Institute rules for winning guarantee that even if you figure out the correct solution today…
  - Realistically, at least a decade will pass before you receive the money
  - The absolute minimum wait is two years
- CMI updated its rules for winning its prizes in 2018. The quick summary of the rules is here:

  https://www.claymath.org/millennium-problems/rules/
- The text of the quick summary is on the next slide
- CMI's definition of a Qualifying Outlet (§6), meaning mostly a respected math journal, is here:

  https://www.claymath.org/wp-content/uploads/2022/03/millennium_prize_rules_0.pdf

Official Summary of CMI Prize Rules

Rules for the Millennium Prize Problems

The revised rules for the Millennium Prize Problems were adopted by the Board of Directors of the Clay Mathematics Institute on September 26, 2018.

Please read this document carefully before contacting CMI about a proposed solution. In particular, please note that:

1. **CMI does not accept direct submission** of proposed solutions.

2. The document is a complete statement of the rules and procedures: **CMI will not offer any further guidance** or advice.

3. Before CMI will consider a proposed solution, **all three of the following conditions must be satisfied**:

   (i) the proposed solution **must be published in a Qualifying Outlet** (see §6), and

   (ii) **at least two years must have passed since publication**, and

   (iii) **the proposed solution must have received general acceptance** in the global mathematics community

Warning #2 from a Nobel Laureate Close to the Problem

- David Gross was the 2004 Physics Nobel Prize co-recipient
- He developed the concept of "asymptotic freedom" of quarks within protons, neutrons, and many other particles
  - Quarks act as if linked by rubber bands (the *real* origin of "string" theory)
  - At close range, the rubber bands goes limp, and the quarks stop attracting
  - His theory annihilated the earlier "physics is pure math" S-matrix theory

"Today's subject is probably the hardest — or at least, that's what Edward Witten tells me — of the seven Millennium Prizes. Now, Arthur Jaffe was an old friend and colleague of mine when he started putting together the Clay Foundation (before he was fired!) He had a brilliant idea of how to get a lot of publicity for the Clay Institute by formulating prizes that would be announced at the beginning of the millennium. Then the Clay Foundation would get a lot of publicity, and arouse a lot of interest in mathematics — and maybe even in these problems — but not have to pay any money. So far, they've been very successful. They've gotten millions of dollars' worth of publicity, and only had to pay out one price to somebody whom, maybe they even knew, wouldn't accept it. So, it's cost them nothing, but it clearly has aroused a lot of interest."

—David Gross, Jan 2, 2018, in "Millennium Prize Problem: Yang Mills Theory" (YouTube)

Warning #3: The Problem is Simpler than it Sounds

- You've likely heard this: Photons have no rest mass (correct)
- You may also have heard this:
  - Every proton and neutron is a triplet of roughly electron-like "quarks"
  - Quarks have unique charges that exchange analogs of photons called "gluons" because they stick the quarks together (the 1970s were… unique)
- Gluons have always been assumed to be massless, like photons
- The real Yang-Mills challenge is: **Prove that gluons have mass**
- **This is a physics problem**, not a math problem!
  - Gluons either have mass or they don't. Reality decides that, not math.
  - The **hope** (Jaffe, Witten, CMI) was that maths could predict gluon mass
  - David Gross broke pure-math S-matrix. He's not a fan of such approaches.
  - He nonetheless provides some of the best advise available on gluon mass

The Actual Wording of the CMI Challenge

Yang–Mills Existence and Mass Gap. Prove that for any compact simple gauge group G, a non-trivial quantum Yang–Mills theory exists on and has a **mass gap $\Delta > 0$**. Existence includes establishing axiomatic properties at least as strong as those cited in Streater & Wightman (1964), Osterwalder & Schrader (1973) and Osterwalder & Schrader (1975).

— A. Jaffe and E. Witten, Quantum Yang-Mills Theory, The Millennium Prize Problems 1, 129–152 (2006). https://www.claymath.org/millennium-problems/yang%E2%80%93mills-and-mass-gap

Warning #4: Be wary when folks overcomplicate simple statements

- The simple physics statement still holds: *Prove gluons have mass*
  - The CMI statements are an educated guess that gluons *may* have mass
  - There are no warnings that the guess could well be completely wrong (!)
  - Another David Gross comment about this challenge applies:

"One of my mentors, Murph Goldberger, at Princeton, wrote … that 'mathematics is an interesting intellectual sport, but it should not be allowed to stand in the way of obtaining sensible information about physical processes.'" **—David Gross, Jan 2, 2018, in "Millennium Prize Problem: Yang Mills Theory" (YouTube)**

Warning #5: The Yang-Mills Paper was Kind of Klutzy

- The reference accorded to Yang-Mills does not match the paper
- Good points of the Yang-Mills paper:
  - The idea that there were new forces inside protons was brilliant!
  - It nicely summarized that neutrons and protons seemed deeply related
  - It identified relationships that, in time, helped produce the Standard Model
- Some subtle bad points about the Yang-Mills paper:
  - It began a trend towards creating redundant descriptive complexity

"It is pointed out that the usual principle of invariance under isotopic spin rotation is not consistent with the concept of localized fields."

  - "Isotopic spin" is an abstract rotation that converts a neutron into a proton
  - The moment you *add a positive electric charge to a neutron*, the new field *trivially* makes it no longer "consistent with the concept of localized fields."
  - Result: The Standard Model consistently overcomplicates electric charge

Part II.
Conventional Approaches

Conventional Approaches to the Problem

Noted mathematician Michael Atiyah presented on it first in 2000:

M. Atiyah, *The Yang-Mills Problem (Millennium Prize Problem II-3)*, The Millennium Prize Problems [May 24] (2000), https://youtu.be/trxjCOmg8GY?t=29m25s

[37:43] Michael Atiyah: … Problem 3 of the Clay Institute list is … to establish Yang-Mills theory as a rigorous quantum field theory. … The important thing is to [prove] the existence of a *positive mass gap*. … The Hamiltonian of the theory's first eigenvalue should be strictly positive. [Physicists believe this] to be true, [so] the [goal] is to establish a rigorous [mathematical] theory with this as a consequence.

… This, then, would provide the mathematical basis for the real physics of the world. It would also pass on, by justification, all the [resulting] beautiful applications, to mathematics. So, both physicists and mathematicians would like to know whether this can be done. …

… This is … a very difficult problem. It has to do with infinite dimensional analysis, because you're dealing with a quantum field theory. It involves analysis, and certainly a lot of topology and algebra, because these spaces are very complicated.

… I … think [this is] the hardest of the [Clay Institute] problems… In some sense, it is the most recent, [though] Yang-Mills equations go back to 1950s. But these attempts to do this are more recent… [38:59]

A One-Hour (!) Overview of the Background Physics

Lorenzo Sadun, *Yang-Mills existence and mass gap* (Video). 2001 University of Texas Lectures on the Millennium Problems (2001).

Full talk: https://youtu.be/6sDJWSjEE8Y?t=08m50s.
Yang-Mills problem: https://youtu.be/6sDJWSjEE8Y?t=57m59s

Very Quickly Covered:

- Yang-Mills Intro
- Charles Coulomb
- James Clark Maxwell
- Maxwells equations
- Planck's constant
- Einstein
- De Broglie
- Schrödinger equations
- Klein-Gordon equation
- curvature
- covariant derivatives
- gauge theory
- quantum mechanics
- nuclear physics
- isospin
- Standard Model
- Yang-Mills Problem

Jaffe-Witten Write-Up (2006, Five Years Later)

A. Jaffe and E. Witten, *Quantum Yang-Mills Theory*, The Millennium Prize Problems **1**, 129–152 (2006).
https://www.claymath.org/millennium-problems/yang%E2%80%93mills-and-mass-gap

Outline

1. The Physics of Gauge Theory
2. Quest for Mathematical Understanding
3. Quantum Fields
4. The Problem
5. Comments
6. Mathematical Perspective
   6.1 Methods
   6.2 The First Examples: Scalar Fields
   6.3 Large Coupling Constant
   6.4 Yukawa Interactions and Abelian Gauge Theory
   6.5 Yang–Mills Theory
   6.6 Further Remarks

David Gross is Easily Closer than Anyone Else

David Gross, *Millennium Prize Problem: Yang Mills Theory* [Video]. Kavli Institute for Theoretical Physics (YouTube), Jan. 2, 2018. https://youtu.be/vMiY7zlBOFI?t=0h00m00s

Despite Gross's annoyance with the Millennium program, his Nobel Laureate background and subsequent work easily place him closer to a meaningful solution than anyone else in the world:

[1:35:40] "By now, **after 30 years or more of doing these calculations — a massive effort** — we have calculated the spectrum of all the low-lying hadron particles, made out of mostly quarks — which I haven't talked about here. But in pure QCD, which we can't compare with experiment, it would be the masses of particles made out of these interacting quanta of the gauge field, and they agree with experiment to about one percent accuracy. There are various ways of estimating theoretical errors as well, **but there's nothing rigorous about this**. One, nobody's ever shown the limit exists, and two, nobody has a rigorous way of controlling these errors. However, we have enough. … Every state that exists in nature appears in the [asymptotic freedom] spectrum. There are no states with *zero* mass; **so, there's a gap, indeed, numerically**. And that's why we believe the answer should be, **yes: this limit exists**, and yes, if one calculates the spectrum of the energy, **there's a gap, and an interesting spectrum**. So, unfortunately, I didn't have time to discuss this remarkable phenomenon [of mass-like insertion into the asymptotic freedom equation], which is the basis of this."

David Gross Will Never Win the Yang-Mills Prize

While David Gross's group has largely proven the gluon rest-mass problem from a physics perspective, the CMI set up the problem very carefully so that only math modeling can win it, not discovery and proof of new physics. This is true even if the physics reaches Nobel Prize levels

> [12:51] "But our [physics] evidence is based on a way of thinking that many mathematicians find difficult to absorb. … Ed Witten and others at the Institute did an experiment ten years ago in Princeton to try to educate … mathematicians in quantum field theory … **David Cosgrove ... rewrote [my] lecture notes, and I couldn't read them — I couldn't understand them** afterward. It was impossible, more or less."
>
> [15:00] "But more interesting, I think, is the feeling that if the mathematics could be developed to answer such questions, that, given the beauty of the [quantum field] theory as physicists see it, those methods, if they were powerful enough to answer these questions, would have to be both beautiful and useful. **But it's clear nobody has really a good idea, as far as I can see, for the mathematical tools. So, the prize is might be useful in pushing mathematicians to develop those tools — and they will probably be very different than anything that's been tried so far.**"
>
> —David Gross, *Millennium Prize Problem: Yang Mills Theory* [Video]. Kavli Institute for Theoretical Physics (YouTube), Jan. 2, 2018. https://youtu.be/vMiY7zlBOFI?t=1h35m40s

If You Are Serious in Pursuing the Yang-Mills Prize…

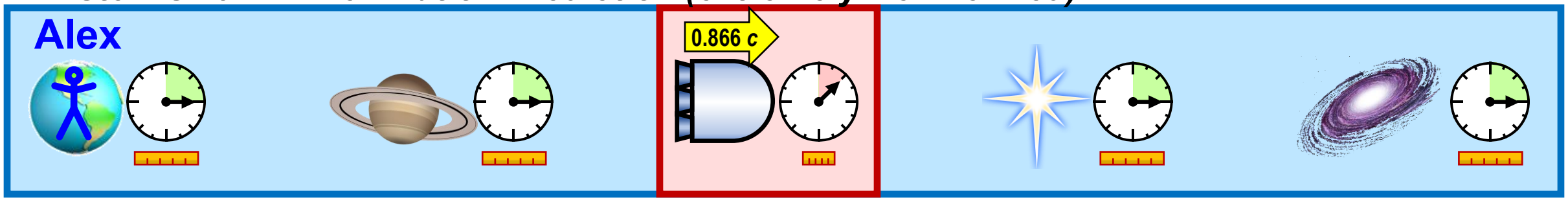
- Then **watch and follow up on every detail of Gross's video**
- Everything else, including CMI materials, are fluff in comparison
- To win the prize, you must **defy the narrow assumptions** of CMI (The only way to win is to realize, like Gross, that you must lose.)
- **Take Gross up on his challenge** to find radically new maths

David Gross: Millennium Prize Problem: Yang Mills Theory (Video). Kavli Institute for Theoretical Physics (YouTube), Jan. 2, 2018. https://youtu.be/vMiY7zlBOFI

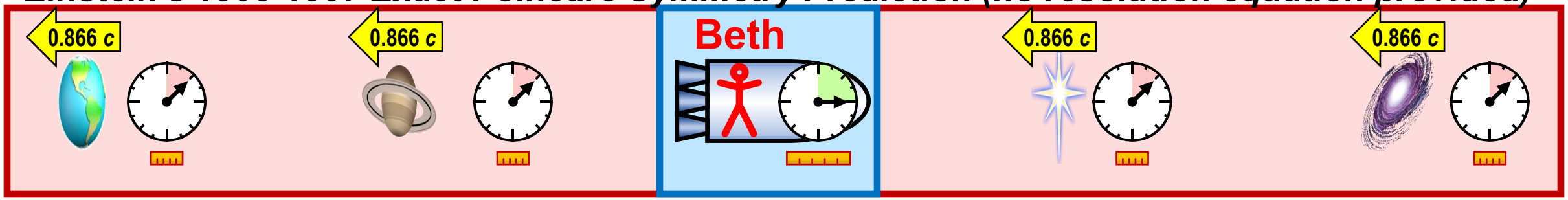
Part III.
Unconventional Approaches

Traditional Symmetry Groups Are Too Simplistic

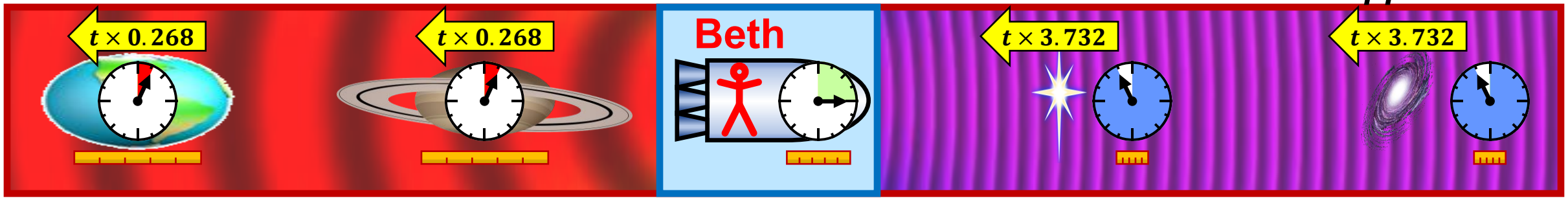
Einstein's 1911 Time-Dilation Prediction (extremely well-verified)



Einstein's 1905-1907 Exact Poincaré Symmetry Prediction (no resolution equation provided)



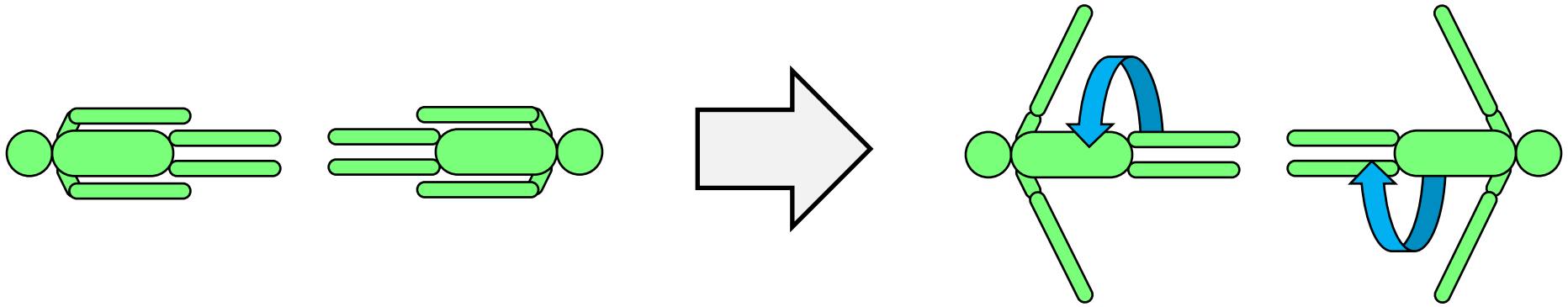
Observed Behavior: Local Matter-Bound Poincaré Bubble in a Vast Relativistic Doppler Sea



The Deep-Math Problem of Ignoring History

- Ignoring history leads to axiomatically incorrect foundations
- Relativity example:
  - Every *real* accelerated system has an action-reaction history
  - Every *real* accelerated system was *explicitly* energized
  - Every *real* accelerated system has a very finite scope of "new" spacetime
- Quantum mechanics example:
  - Every example of a delocalized wave function *begins* classically
  - The concept of a "universal" wave function ignores such beginnings
- Spacetime continuums
  - Einstein before 1911 used only clocks and rulers to measure spacetime
  - Ignoring actual instruments leads inevitably to non-quantum relativity

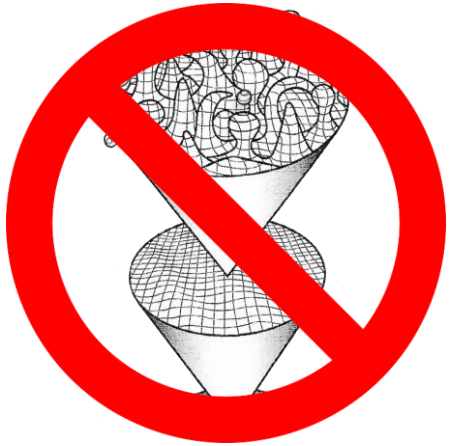
Another Example: Remembering the Past and Mach's Principle

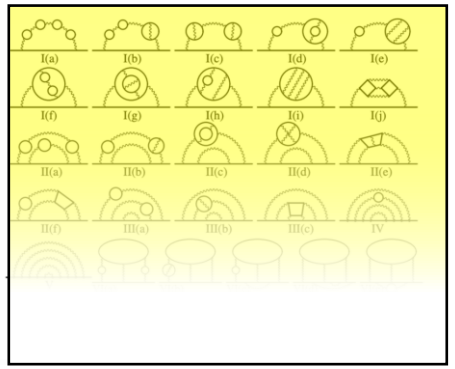


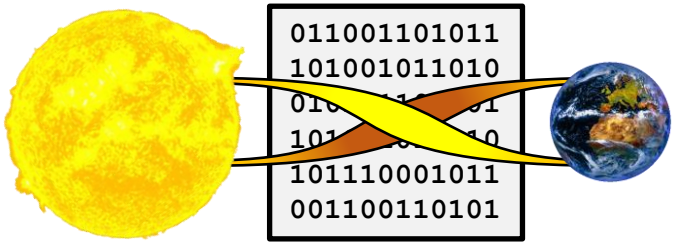
- A surprising number of "beautiful" maths in physics, especially symmetries, obtain their beauty by *forgetting or ignoring the past*
- Forgetting the past is like eating the center out of the pie: You get perfection only by *pretending* no larger context exists
- For Mach's Principle, looking only at an object's spin *now* requires pretending it has no past history with the rest of the universe
- The needed Mach link to the past is ***angular momentum conservation***
- The implications of "**localized Mach memory**" extend down to particle spin

A Data-Driven Idea: Sparse-Information Physics

- Only matter can create space and time; the vacuum is truly empty
  - "Empty" is fully compatible with special relativity (and *centuries of thought*)
  - No Planck foam, no QCD fluctuations *except* when energized
  - Quantum fields become *localized* by inertial frames and "funded" by energy
  - No "effective" field theories (no cut-offs, no vacuum density issues)
  - Gravity and motion become *encodings* of relationships between objects







The Hilbert Paradox

Hilbert's Sixth Problem:
6. Mathematical Treatment of the Axioms of Physics. The investigations on the foundations of geometry suggest the problem: *To treat in the same manner, by means of axioms, those physical sciences in which already today mathematics plays an important part; in the first rank are the theory of probabilities and mechanics.*

- Hilbert relies on axioms that assume:
  - Symmetry groups such as Poincaré have no limits in the physical universe
    This is not true for any actual instance of a Poincaré group
  - Points are real and independent of the coordinate frame of the observer
    This is not true for any system existing in the physical universe
  - Continuums and smooth manifolds are fundamental givens
    This is not true for any physical or computed entity (infinities abound)
- **The Hilbert Paradox**: *One must abandon Hilbert to solve Hilbert*

The Yang-Mills Generalization of Maxwell

M. Atiyah, *The Yang-Mills Problem (Millennium Prize Problem II-3), The Millennium Prize Problems*, May 24, 2000.

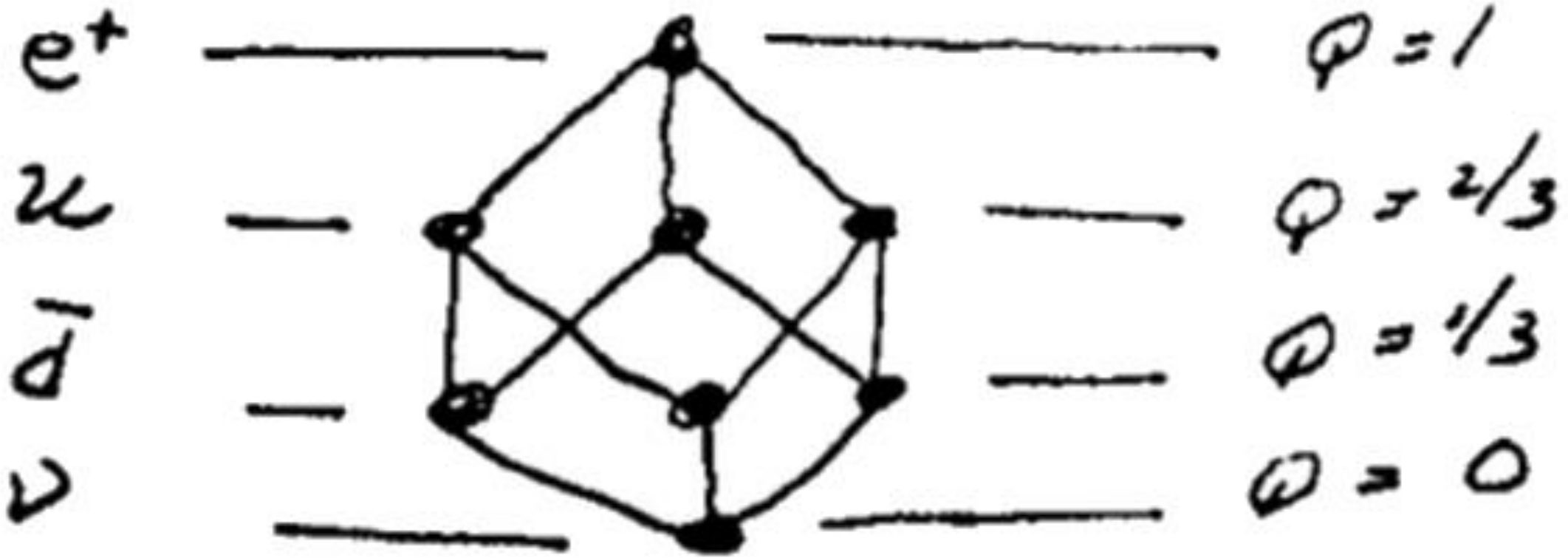
Video: https://youtu.be/trxjCOmg8GY?t=29m25s.

Transcript: https://sarxiv.org/ref.2000-05-24.pdf

[32:56] "Now, going beyond quantum electrodynamics, we come into the 1950s, where the Yang-Mills equations were introduced. **The Yang-Mills equations are, very crudely speaking, a matrix version of Maxwell's equations**. You take eight Maxwell's equations, looked at it in the right way, and you generalize them to matrices. And then, you get a new set of equations called the Yang-Mills equations, and then you treat all the previous theory with an extra complication."

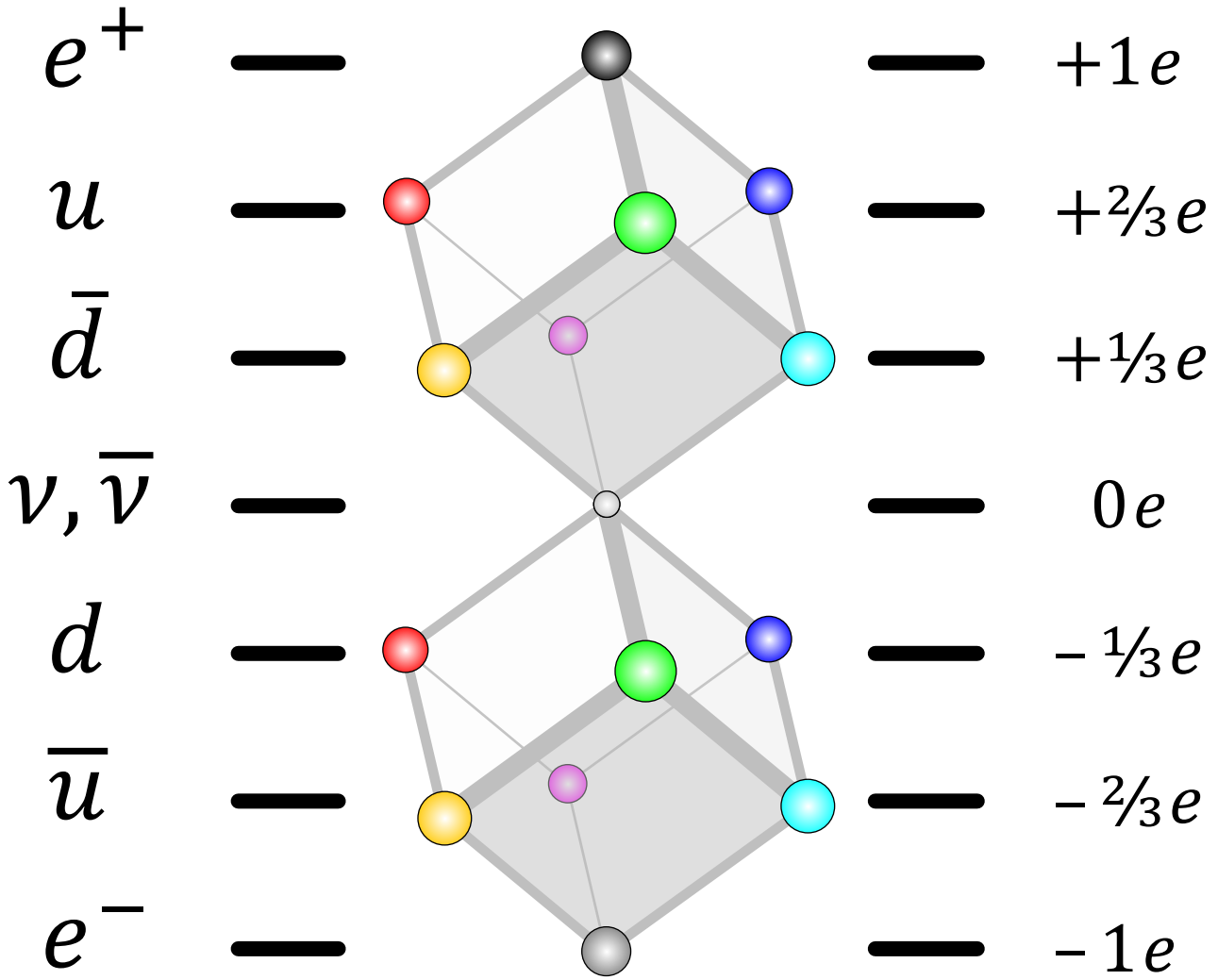
- **There are other ways to generalize Maxwell's equations**
  - What does the actual particle data say about Maxwell's "displacements"?
  - Can such models produce simpler ways to generalize particle models?

The Glashow Fermion Cube



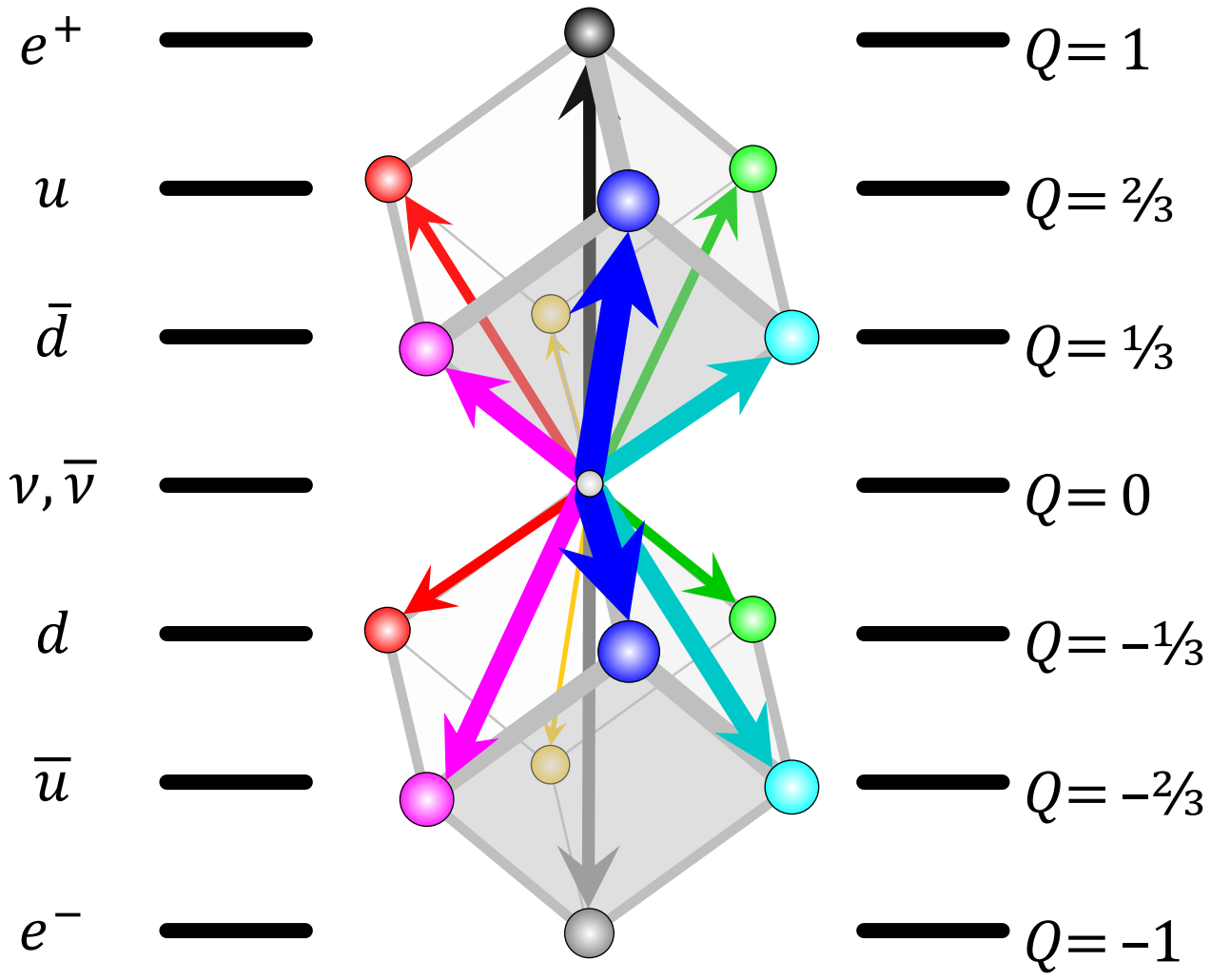
S. L. Glashow, "The Future of Elementary Particle Physics [HUTP-79/A059]," *Harvard University Preprints*, Jul. 1979. Available: https://inspirehep.net/literature/144466. Page 29, Section III, *Let the Desert Bloom!* S. L. Glashow's original hand drawing.

Adding the Second Glashow Cube

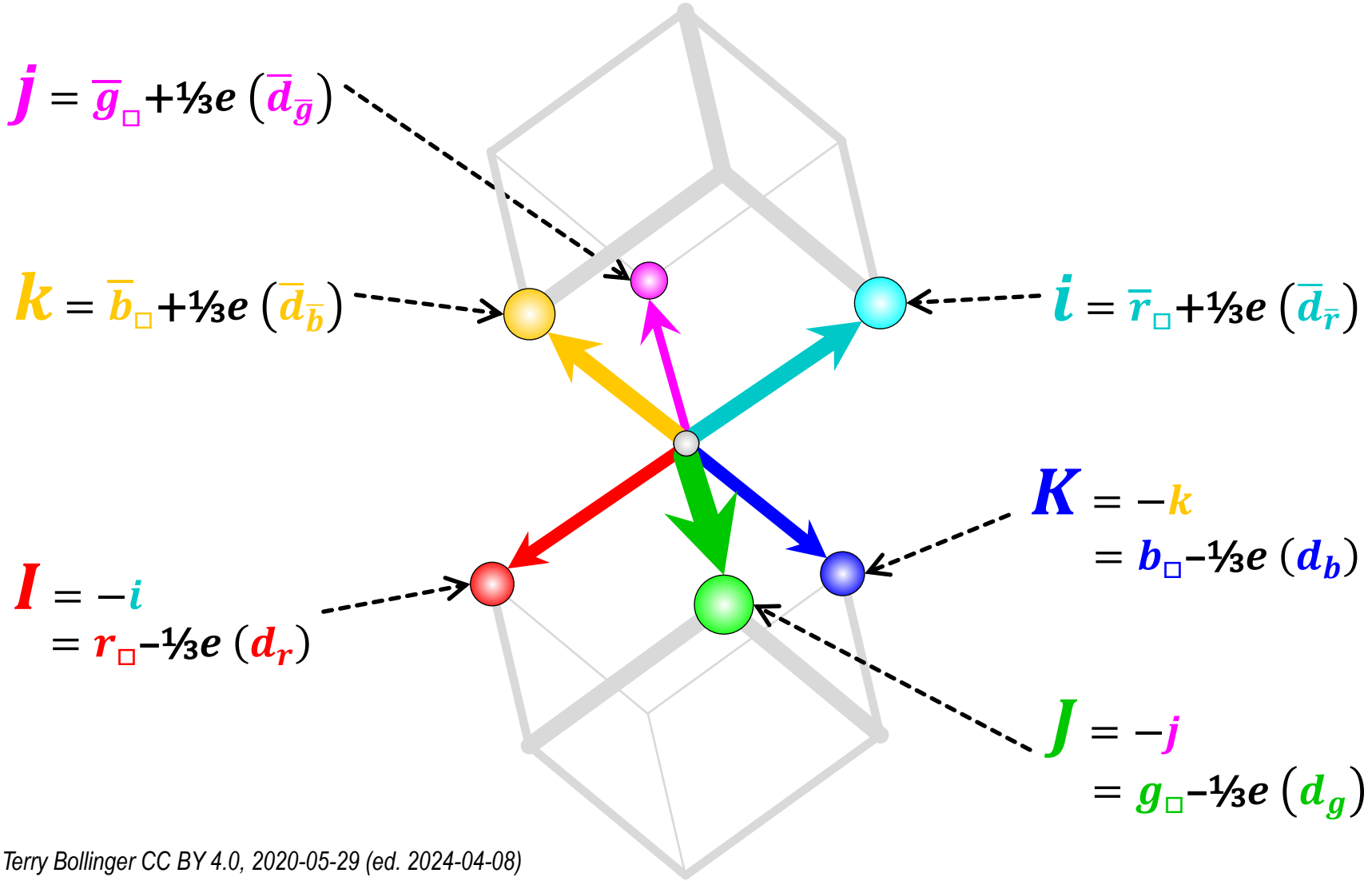


Terry Bollinger CC BY 4.0, 2020-05-29 (ed. 2024-03-29)

Strong-Electric Glashow Vectors = 3D Maxwell

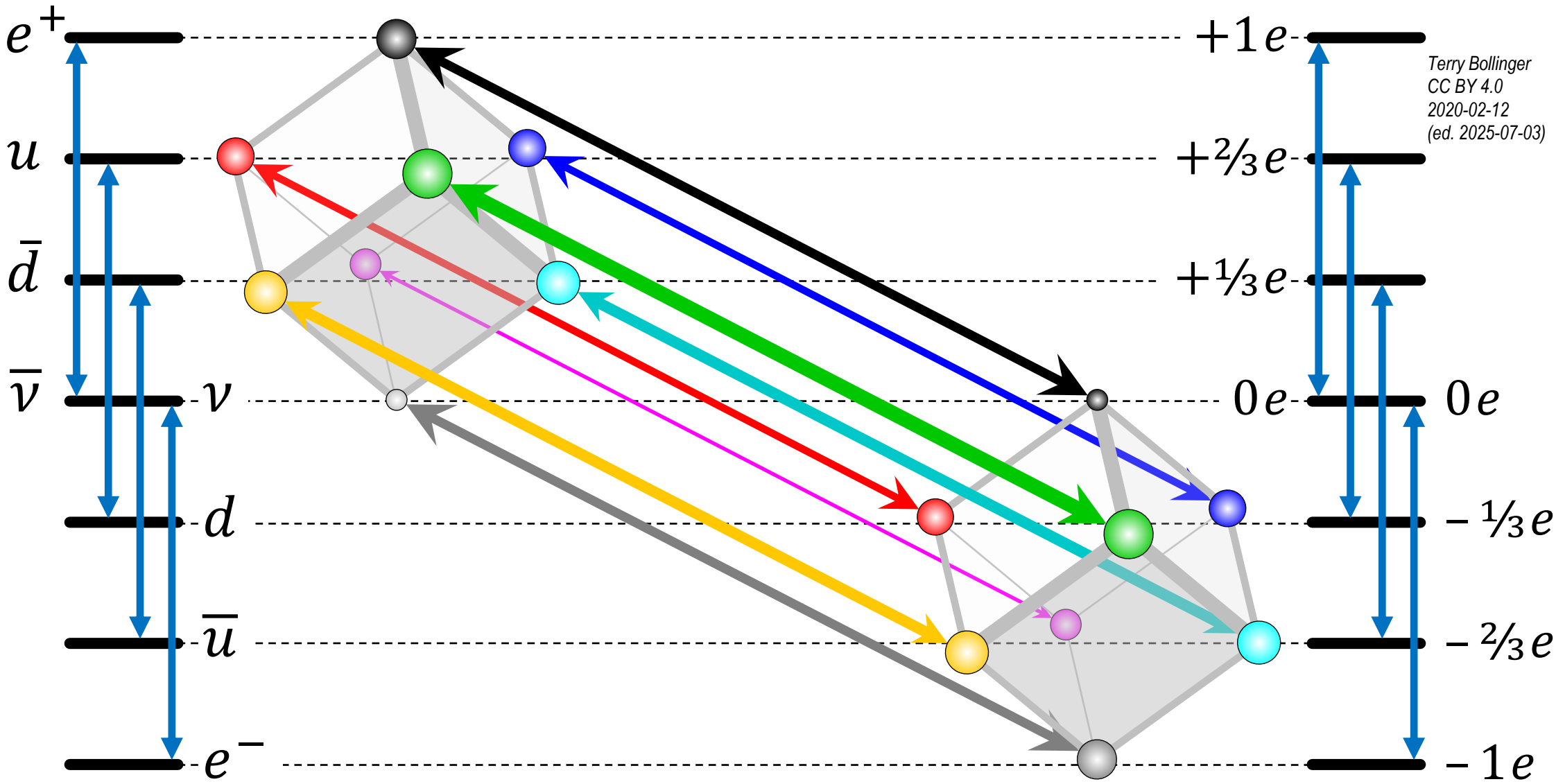


The Six Glashow Unit-Charge Vectors

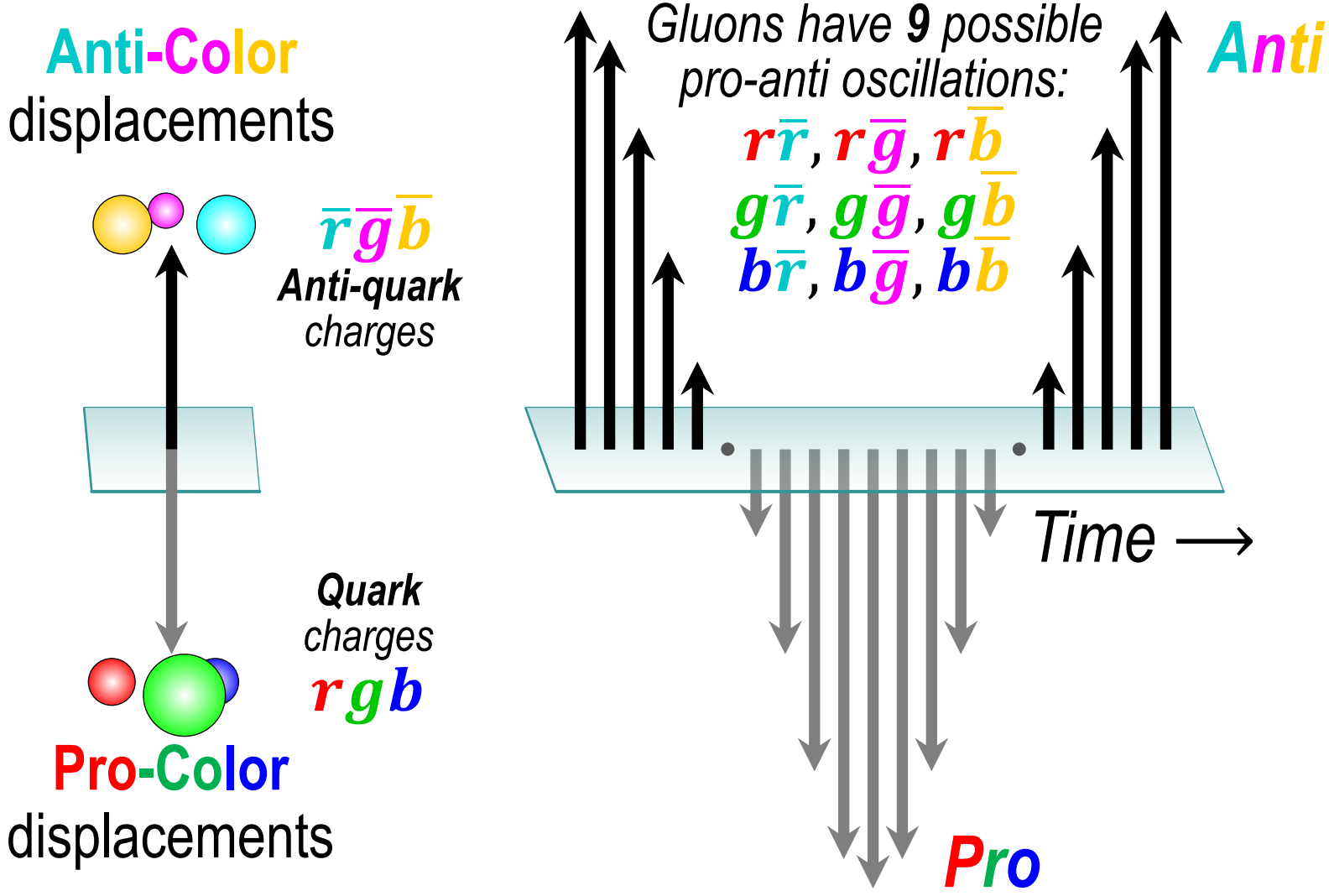


Terry Bollinger CC BY 4.0, 2020-05-29 (ed. 2024-04-08)

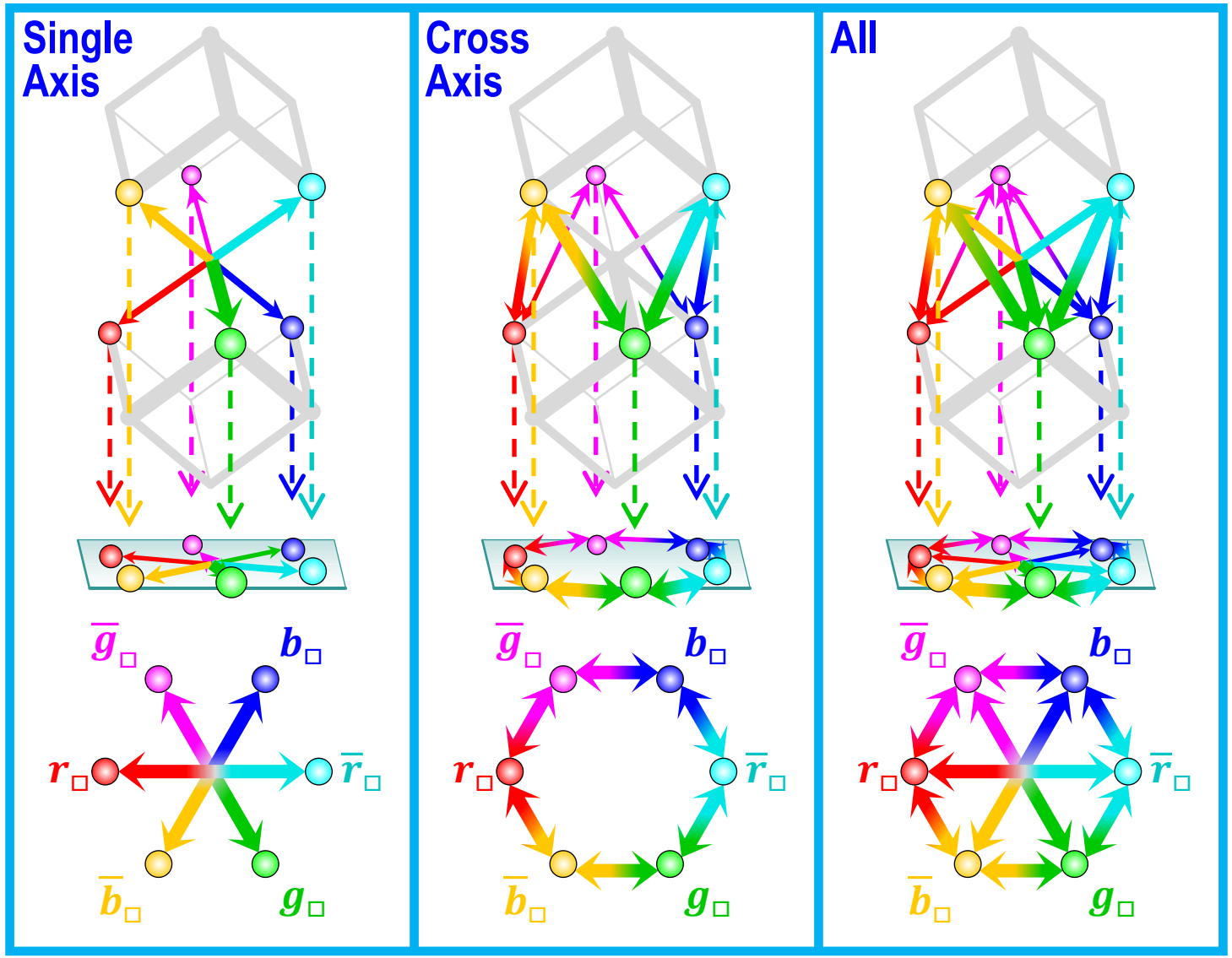
$SU(2)$: Gen-1 Fermions as T_3 Weak-Isospin Tesseract Bridge Vectors



Gluons as Pro-Anti Color Oscillations [Added]



Gluons as Non-Vertical 3-Space Oscillations [Added]



Glashow Tesseracts and Yang-Mills [Edited]

- Each bridge vector captures the Yang-Mills "isospin" concept
  - No false distinctions: The fourth dimension is about electric charge
  - The unified 3D Maxwell + Color Euclidean space simplifies the relationship between charge displacement, time, and mass
- Hypothesis: **Time and charge geometries link closely**
  - **Only** the electric charge fully aligns with local time
  - Any particle outside the central axis **has non-Higgs mass**
  - This **applies to charge oscillations (gluons)**, as well as fermions
  - The result is a **unified geometric approach to time and mass**
  - The photon becomes the missing 9th (symmetric) gluon sum

Summary

- Pay the most attention to David Gross, not CMI
- Work to solve the problem, not to get the prize
  - The CMI prize likely will never materialize
  - You might well get a Nobel Prize, however
- Bottom-up “sparse information” methods simplify Yang-Mills
  - Beware of math noise from past sloppy assumptions (e.g., isospin)
  - Infinities are always bad. The universe *never* works that way
- Grand unification is a bogus goal; electric and color *never split*
  - This single misconception has impeded physics for decades