It's Time to Reexamine Euclid's Definition of a Point

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https://youtu.be/lFlu60qs7_4&lc=UgzaF5ncpUTIeDWCizZ4AaABAg

A Comment on the Veritasium (Derek Muller, YouTube) post:
How One Line in the Oldest Math Text Hinted at Hidden Universes (Oct 21, 2023)
https://youtu.be/lFlu60qs7_4

Conclusion: Based on modern physics, it's time to reexamine Euclid's definition of a point.

17:26 AK *"[Euclid's] Definition 1: 'A point is that which has no part.' What does it mean to have a part? What is a part? What does it mean not to have a part?"*

18:58 DM *"You can think of geometry as a game. The first four postulates, [which in turn are based on 23 definitions,] are like the minimum rules required to play."*

Given modern scientific knowledge, Alex Kontorovich and Derek Muller, why not take this game all the way down to Definition 1, which unavoidably touches on the quantum world?

Euclid's intuition in Definition 1 is not hard to fathom: He's thinking of a point as what you get if you shrink a ball until it has no detectable volume — a ball with no "parts." A cube might also have worked, but if you think about it, it has parts: corners and faces that give it unique orientations. A cylinder might also have worked, but that's even worse since the result is not a point but what most would call a line segment. A ball works best, not because it's completely featureless — it does have a surface — but because it's arguably the natural form with the fewest possible distinct parts. Euclid's use of "that" was likely a bit of cover, a reluctance to admit that a ball was the only "that" he found acceptable.

So, using more precise modern terminology, what was Euclid trying to say with his intuitive definition of a point? It was a calculus problem: He proposed that the most straightforward definition was the infinitesimal down-scaling of a three-dimensional ball.

This degree of complexity should trigger alarms. If a point is the simplest and most fundamental definition in all of geometry, why did Euclid define it using what we would now call a three-dimensional calculus limit, applied to a specific category of classical objects noted for stripping complexity down to a minimum?

No matter!

I mean that literally: You can't use matter. Euclid did not know — could not have known — that there is *no version of matter* capable of reaching his point limit. The uncertainty principle blew all of that away in the early 1900s, causing the tiniest of material objects, such as electrons, to *expand,* not shrink, if you take away too many "parts" of their mass. Paradoxically, one must go in the *opposite* direction to get more point-like: You must *add* more parts — add more mass — to push farther down into Planck's see-saw-like size limit.

Muons, for example, are hundreds of times more point-like than electrons, but only at the cost of adding about 200 more "parts" of energy. It gets much worse. If made of matter, the creation of only one literal Euclidean point would require adding more energy than exists in the universe. That's a lot of added parts!

So again: *No matter!*

The point is that unquestioning acceptance of Euclid's definition of points requires abandoning the world of matter and energy that inspired it. While removing parts is a simple, low-energy, and nicely linear process for the entirety of classical reality as Euclid knew it, that same removal process fails spectacularly at the scale of atoms. Objects at that tiny scale — one far smaller than anything available to Euclid — begin objecting strenuously and energetically to having additional parts stripped off.

Dr. Muller's video details how exploring Euclid's Fifth Postulate made Riemannian manifolds possible. That, in turn, gave Einstein a framework for defining the universe's large-scale structure in ways impossible using Euclidean geometry. Yet, paradoxically, that same Riemannian geometry remains as rigorously true to Euclid's twenty-three Definitions and first four Postulates of Euclid as Euclidean geometry.

General Relativity is incompatible with quantum mechanics due to the *geometry* it uses, not because it favors curved space over quantum force particles. Euclidean and non-Euclidean geometries *equally* reject the inclusion of matter in their shared first definition. The irony is profound, given that General Relativity is about how matter bends spacetime.

It's time to fix this. Just as General Relativity required a deeper, more flexible look at the Fifth Postulate, the merger of General Relativity and quantum mechanics requires a deeper, more flexible look at the simplest of all concepts: What is a "point"?

Look at Euclid's Definition 1 from a modern perspective of calculus and awareness of the role of energy at small scales. Instead of absolutes, think of his definition as a variable-depth *process* that strives for, but never quite achieves, a ball with no parts. Instead of absolutes, look for dynamics. Instead of assuming a point as a trivial given, look for a process that repeats and converges, with a complexity more comparable to hydrogen.

Conservation of energy is a huge clue. For Euclid, energy was conveniently pre-packaged into matter — into rocks, sand, and dust — that lent easy credence to removing parts until no parts are left. Millennia later, Einstein figured out that matter is, in fact, merely energy in an especially persistent and predictable form. This persistence fooled Euclid into thinking energy played no role in his definitions. We, however, do not live in that world. Particle accelerators tell us in excruciating detail how mutable energy becomes when you break matter down below the deceptively simple point approximations of atoms.

Frustratingly, another profound clue for updating points is the baffling nature of quantum observation. Literally by definition, a Euclidean point can never be in two locations at once. However, this simplicity does not exist in the complex dynamics of material near-points. The paradox of Feynman's double-slit experiment is not whether a point-like particle can pass through two slits. It's in what we *mean* by saying a particle is a "point."

As nicely described in Dr. Muller's video, brilliant people over two millennia devoted entire careers to pondering the excessive specificity of Euclid's Fifth Postulate. That brings up a different question: How many people have pondered the need for a better point definition?

Among academic professionals — a category in which I emphatically play no part and so omit myself from the count — Professor Alex Kontorovich, in this video, is the only person I recall seeing consider the issue. I'm sure others have considered it informally. However, if you are in academia, do you truly want to risk your tenure (you easily could) by publicly declaring that the very first definition of Euclid's Elements is naïve and needs a massive overhaul that would replace the very concept of points with variable-scale convergent processes that would *necessarily* defy our classical concepts of space and time?

Nonetheless, if the goal is General Relativity and quantum theory unification, *it matters*.