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Special Relativity Conversions with Relativistic Doppler, Binary Rapidity, and Age Gradients

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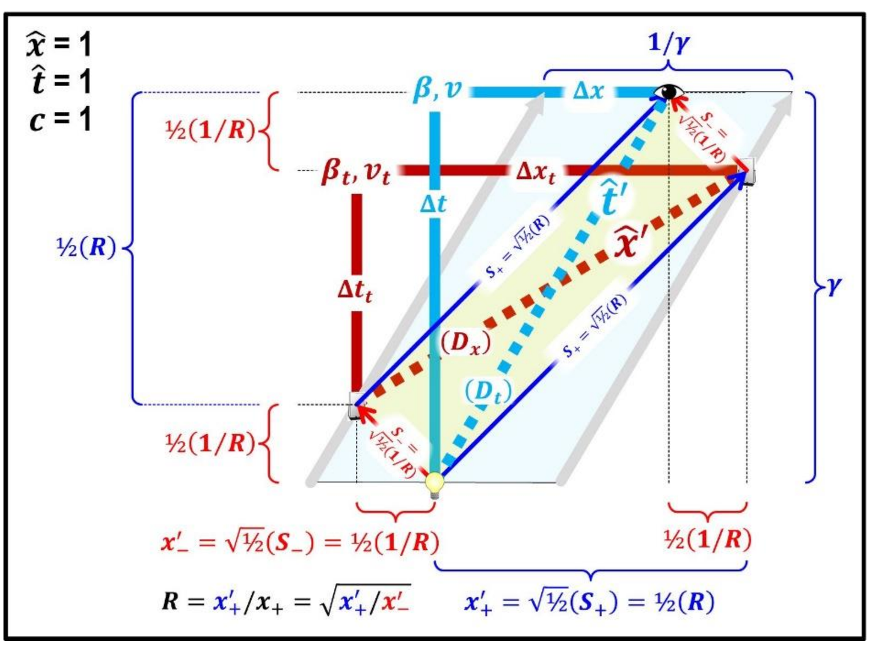
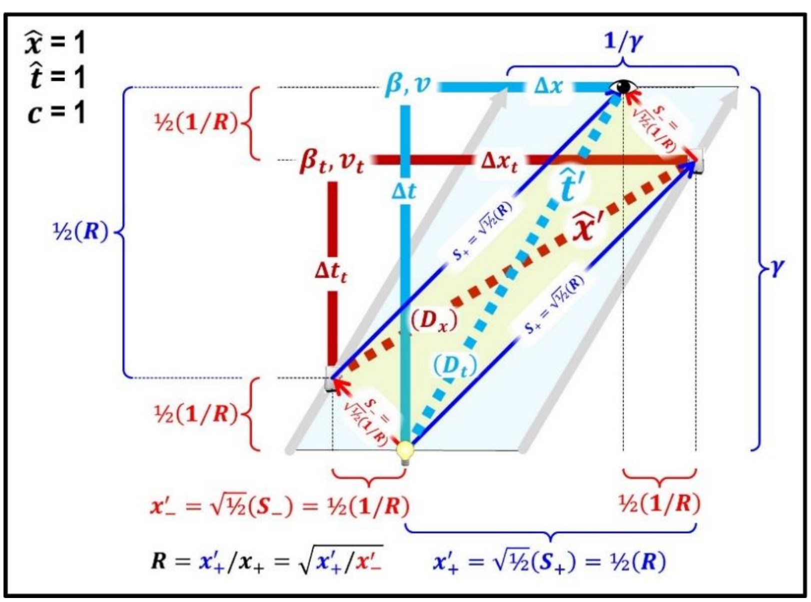
(2025-07-27: α' fixes, refactored)

	Velocity $v =$	Unitless Velocity $\beta =$	Lorentz Factor $\gamma =$	Rapidity $w =$	Binary Rapidity $\rho =$	Relativistic Doppler Factor (Forward Light Paths Ratio) $R =$	Age Gradient $\alpha =$	Traveler's Gradient $\alpha_+ =$	In-Frame Gradient $\alpha' =$	Diagonal Factor $D =$
Best→	v	$\frac{v}{c}$	$\frac{R+R^{-1}}{2}$	$\ln R$	$\log_2 R$	$\sqrt{\frac{1+\beta}{1-\beta}}$	$-\frac{\beta\gamma}{c}$	$-\alpha$	$-\frac{v}{c^2}$	$\sqrt{2\gamma^2-1}$
Given↓ v	v	$(v)c^{-1}$	$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$	$\ln\sqrt{\frac{c+v}{c-v}}$	$\log_2\sqrt{\frac{c+v}{c-v}}$	$\sqrt{\frac{c+v}{c-v}}$	$\left(-\frac{v}{\sqrt{c^2-v^2}}\right)c^{-1}$	$\left(\frac{v}{\sqrt{c^2-v^2}}\right)c^{-1}$	$\left(-\frac{v}{c}\right)c^{-1}$	$\sqrt{\frac{c^2+v^2}{c^2-v^2}}$
β	$c(\beta)$	β	$\frac{1}{\sqrt{1-\beta^2}}$	$\ln\sqrt{\frac{1+\beta}{1-\beta}}$	$\log_2\sqrt{\frac{1+\beta}{1-\beta}}$	$\sqrt{\frac{1+\beta}{1-\beta}}$	$\left(-\frac{\beta}{\sqrt{1-\beta^2}}\right)c^{-1}$	$\left(\frac{\beta}{\sqrt{1-\beta^2}}\right)c^{-1}$	$(-\beta)c^{-1}$	$\sqrt{\frac{1+\beta^2}{1-\beta^2}}$
γ	$c\left(\sqrt{1-\frac{1}{\gamma^2}}\right)$	$\sqrt{1-\frac{1}{\gamma^2}}$	γ	$\ln\sqrt{\frac{\gamma+\sqrt{\gamma^2-1}}{\gamma-\sqrt{\gamma^2-1}}}$	$\log_2\sqrt{\frac{\gamma+\sqrt{\gamma^2-1}}{\gamma-\sqrt{\gamma^2-1}}}$	$\sqrt{\frac{\gamma+\sqrt{\gamma^2-1}}{\gamma-\sqrt{\gamma^2-1}}}$	$\left(-\sqrt{\gamma^2-1}\right)c^{-1}$	$\left(\sqrt{\gamma^2-1}\right)c^{-1}$	$\left(-\sqrt{1-\frac{1}{\gamma^2}}\right)c^{-1}$	$\sqrt{2\gamma^2-1}$
w	$c\left(\frac{e^w-e^{-w}}{e^w+e^{-w}}\right)$	$\frac{e^w-e^{-w}}{e^w+e^{-w}}$	$\frac{e^w+e^{-w}}{2}$	w	$\frac{w}{\ln 2}$	e^w	$\left(-\frac{e^w-e^{-w}}{2}\right)c^{-1}$	$\left(\frac{e^w-e^{-w}}{2}\right)c^{-1}$	$\left(-\frac{e^w-e^{-w}}{e^w+e^{-w}}\right)c^{-1}$	$\sqrt{\frac{e^{2w}+e^{-2w}}{2}}$
ρ	$c\left(\frac{2^\rho-2^{-\rho}}{2^\rho+2^{-\rho}}\right)$	$\frac{2^\rho-2^{-\rho}}{2^\rho+2^{-\rho}}$	$\frac{2^\rho+2^{-\rho}}{2}$	$\rho\ln 2$	ρ	2^ρ	$\left(-\frac{2^\rho-2^{-\rho}}{2}\right)c^{-1}$	$\left(\frac{2^\rho-2^{-\rho}}{2}\right)c^{-1}$	$\left(-\frac{2^\rho-2^{-\rho}}{2^\rho+2^{-\rho}}\right)c^{-1}$	$\sqrt{\frac{2^{2\rho}+2^{-2\rho}}{2}}$
R	$c\left(\frac{R-R^{-1}}{R+R^{-1}}\right)$	$\frac{R-R^{-1}}{R+R^{-1}}$	$\frac{R+R^{-1}}{2}$	$\ln R$	$\log_2 R$	R	$\left(-\frac{R-R^{-1}}{2}\right)c^{-1}$	$\left(\frac{R-R^{-1}}{2}\right)c^{-1}$	$\left(-\frac{R-R^{-1}}{R+R^{-1}}\right)c^{-1}$	$\sqrt{\frac{R^2+R^{-2}}{2}}$
α	$c\left(-\frac{\alpha c}{\sqrt{1+\alpha^2c^2}}\right)$	$-\frac{\alpha c}{\sqrt{1+\alpha^2c^2}}$	$\sqrt{1+\alpha^2c^2}$	$\ln\sqrt{\frac{\sqrt{1+\frac{1}{\alpha^2c^2}}+1}{\sqrt{1+\frac{1}{\alpha^2c^2}}-1}}$	$\log_2\sqrt{\frac{\sqrt{1+\frac{1}{\alpha^2c^2}}+1}{\sqrt{1+\frac{1}{\alpha^2c^2}}-1}}$	$\sqrt{\frac{\sqrt{1+\frac{1}{\alpha^2c^2}}+1}{\sqrt{1+\frac{1}{\alpha^2c^2}}-1}}$	α	$-\alpha$	$\frac{\alpha}{\sqrt{1+\alpha^2c^2}}$	$\sqrt{1+2\alpha^2c^2}$
α_+	$c\left(\frac{\alpha_+c}{\sqrt{1+\alpha_+^2c^2}}\right)$	$\frac{\alpha_+c}{\sqrt{1+\alpha_+^2c^2}}$	$\sqrt{1+\alpha_+^2c^2}$	$\ln\sqrt{\frac{\sqrt{1+\frac{1}{\alpha_+^2c^2}}+1}{\sqrt{1+\frac{1}{\alpha_+^2c^2}}-1}}$	$\log_2\sqrt{\frac{\sqrt{1+\frac{1}{\alpha_+^2c^2}}+1}{\sqrt{1+\frac{1}{\alpha_+^2c^2}}-1}}$	$\sqrt{\frac{\sqrt{1+\frac{1}{\alpha_+^2c^2}}+1}{\sqrt{1+\frac{1}{\alpha_+^2c^2}}-1}}$	$-\alpha_+$	α_+	$-\frac{\alpha_+}{\sqrt{1+\alpha_+^2c^2}}$	$\sqrt{1+2\alpha_+^2c^2}$
α'	$c(-\alpha'c)$	$-\alpha'c$	$\frac{1}{\sqrt{1+\alpha'^2c^2}}$	$\ln\sqrt{\frac{1-\alpha'c}{1+\alpha'c}}$	$\log_2\sqrt{\frac{1-\alpha'c}{1+\alpha'c}}$	$\sqrt{\frac{1-\alpha'c}{1+\alpha'c}}$	$\frac{\alpha'}{\sqrt{1-\alpha'^2}}$	$-\frac{\alpha'}{\sqrt{1-\alpha'^2}}$	α'	$\sqrt{\frac{1+\alpha'^2c^2}{1-\alpha'^2c^2}}$
D	$c\left(\sqrt{\frac{D^2-1}{D^2+1}}\right)$	$\sqrt{\frac{D^2-1}{D^2+1}}$	$\sqrt{\frac{D^2+1}{2}}$	$\ln\sqrt{\frac{1+\sqrt{\frac{D^2-1}{D^2+1}}}{1-\sqrt{\frac{D^2-1}{D^2+1}}}}$	$\log_2\sqrt{\frac{1+\sqrt{\frac{D^2-1}{D^2+1}}}{1-\sqrt{\frac{D^2-1}{D^2+1}}}}$	$\sqrt{\frac{1+\sqrt{\frac{D^2-1}{D^2+1}}}{1-\sqrt{\frac{D^2-1}{D^2+1}}}}$	$\left(-\sqrt{\frac{D^2-1}{2}}\right)c^{-1}$	$\left(\sqrt{\frac{D^2-1}{2}}\right)c^{-1}$	$\left(-\sqrt{\frac{D^2-1}{D^2+1}}\right)c^{-1}$	D



All special relativity velocity factors are derivable using a light clock with two mirrors equidistant from a light-pulse emitter/detector. The distance between mirrors defines one length unit $\hat{x}$, and the time from pulse emission to return defines one time unit $\hat{t}$. Motion of the system along the axis of mirror separation requires *physical* repositioning (Bell's ship paradox) of the parts in the observer frame to ensure synchronous pulse returns. The new lengths then define the various velocity factors.

	Velocity v V =	*Unitless Velocity* β B =	*Lorentz Factor* γ Y =	*Rapidity* w W =	*Binary Rapidity* ρ P =	*Relativistic Doppler Factor (Forward Light Paths Ratio)* R R =	*Age Gradient* α A =	*Traveler's Gradient* α_+ F =	*In-Frame Gradient* α' J =	*Diagonal Factor* D D =
Best→	V	(V)/c	((R)+((R)^(-1)))/2	ln(R)	log_2(R)	√((1+(B))/(1-(B)))	-((B)(Y))/c	-(A)	-(V)/(c^2)	√(2((Y)^2)-1)
Given↓										
v: **V**	V	((V))(c^(-1))	1/√(1-((V)^2)/(c^2))	ln(√((c+(V))/(c-(V))))	log_2(√((c+(V))/(c-(V))))	√((c+(V))/(c-(V)))	(-(V)/√(c^2-((V)^2)))(c^(-1))	((V)/√(c^2-((V)^2)))(c^(-1))	(-(V)/c)(c^(-1))	√((c^2+((V)^2))/(c^2-((V)^2)))
β: **B**	c((B))	B	1/√(1-((B)^2))	ln(√((1+(B))/(1-(B))))	log_2(√((1+(B))/(1-(B))))	√((1+(B))/(1-(B)))	(-(B)/√(1-((B)^2)))(c^(-1))	((B)/√(1-((B)^2)))(c^(-1))	(-(B))(c^(-1))	√((1+((B)^2))/(1-((B)^2)))
γ: **Y**	c(√(1-(1/((Y)^2))))	√(1-(1/((Y)^2)))	Y	ln(√(((Y)+√(((Y)^2)-1))/((Y)-√(((Y)^2)-1))))	log_2(√(((Y)+√(((Y)^2)-1))/((Y)-√(((Y)^2)-1))))	√(((Y)+√(((Y)^2)-1))/((Y)-√(((Y)^2)-1)))	(-√(((Y)^2)-1))(c^(-1))	(√(((Y)^2)-1))(c^(-1))	(-√(1-(1/((Y)^2))))(c^(-1))	√(2((Y)^2)-1)
w: **W**	c(((e^(W))-(e^(-(W))))/((e^(W))+(e^(-(W)))))	((e^(W))-(e^(-(W))))/((e^(W))+(e^(-(W))))	((e^(W))+(e^(-(W))))/2	W	(W)/ln(2)	(e^(W))	(-((e^(W))-(e^(-(W))))/2)(c^(-1))	(((e^(W))-(e^(-(W))))/2)(c^(-1))	(-((e^(W))-(e^(-(W))))/((e^(W))+(e^(-(W)))))(c^(-1))	√((e^2(W)+e^(-2(W)))/2)
ρ: **P**	c(((2^(P))-(2^(-(P))))/((2^(P))+(2^(-(P)))))	((2^(P))-(2^(-(P))))/((2^(P))+(2^(-(P))))	((2^(P))+(2^(-(P))))/2	(P) ln(2)	P	(2^(P))	(-((2^(P))-(2^(-(P))))/2)(c^(-1))	(((2^(P))-(2^(-(P))))/2)(c^(-1))	(-((2^(P))-(2^(-(P))))/((2^(P))+(2^(-(P)))))(c^(-1))	√((2^2(P)+2^(-2(P)))/2)
R: **R**	c(((R)-((R)^(-1)))/((R)+((R)^(-1))))	((R)-((R)^(-1)))/((R)+((R)^(-1)))	((R)+((R)^(-1)))/2	ln((R))	log_2((R))	R	(-((R)-((R)^(-1)))/2)(c^(-1))	(((R)-((R)^(-1)))/2)(c^(-1))	(-((R)-((R)^(-1)))/((R)+((R)^(-1))))(c^(-1))	√(((R)^2+(R)^(-2))/2)
α: **A**	c(-(A)c/√(1+((A)^2)(c^2)))	-(A)c/√(1+((A)^2)(c^2))	√(1+((A)^2)(c^2))	ln(√((√(1+(1/(((A)^2)(c^2))))+1)/(√(1+(1/(((A)^2)(c^2))))-1)))	log_2(√((√(1+(1/(((A)^2)(c^2))))+1)/(√(1+(1/(((A)^2)(c^2))))-1)))	√((√(1+(1/(((A)^2)(c^2))))+1)/(√(1+(1/(((A)^2)(c^2))))-1))	A	-(A)	(A)/√(1+((A)^2)(c^2))	√(1+2((A)^2)(c^2))
α_+: **F**	c(((F)c)/√(1+((F)^2)(c^2)))	((F)c)/√(1+((F)^2)(c^2))	√(1+((F)^2)(c^2))	ln(√((√(1+(1/(((F)^2)(c^2))))+1)/(√(1+(1/(((F)^2)(c^2))))-1)))	log_2(√((√(1+(1/(((F)^2)(c^2))))+1)/(√(1+(1/(((F)^2)(c^2))))-1)))	√((√(1+(1/(((F)^2)(c^2))))+1)/(√(1+(1/(((F)^2)(c^2))))-1))	-(F)	F	-(F)/√(1+((F)^2)(c^2))	√(1+2((F)^2)(c^2))
α': **J**	c(-(J)c)	-(J)c	1/√(1+(J)^2(c^2))	ln(√((1-(J)c)/(1+(J)c)))	log_2(√((1-(J)c)/(1+(J)c)))	√((1-(J)c)/(1+(J)c))	((J))/√(1-(J)^2)	-((J))/√(1-(J)^2)	J	√((1+(J)^2(c^2))/(1-(J)^2(c^2)))
D: **D**	c(√((((D)^2)-1)/(((D)^2)+1)))	√((((D)^2)-1)/(((D)^2)+1))	√((((D)^2)+1)/2)	ln(√((1+√((((D)^2)-1)/(((D)^2)+1)))/(1-√((((D)^2)-1)/(((D)^2)+1)))))	log_2(√((1+√((((D)^2)-1)/(((D)^2)+1)))/(1-√((((D)^2)-1)/(((D)^2)+1)))))	√((1+√((((D)^2)-1)/(((D)^2)+1)))/(1-√((((D)^2)-1)/(((D)^2)+1))))	(-√((((D)^2)-1)/2))(c^(-1))	(√((((D)^2)-1)/2))(c^(-1))	(-√((((D)^2)-1)/(((D)^2)+1)))(c^(-1))	D



The table above has text versions of equations that are compatible with Google's equation processor. After replacing the capital letters with numeric values, they can be pasted directly into a Google prompt to get numeric results.

Notes: The forward light path ratio R gives a geometric interpretation of the relativistic Doppler factor. Binary rapidity is easier to use in figures than standard rapidity. Age gradients quantify the rate of non-simultaneity per length of objects, e.g., Einstein's trains.