

|                                   |
|-----------------------------------|
| <b>SFL Volume 39 (2026-Q1-Q4)</b> |
|-----------------------------------|

---

[2026-01-07.12:25 EST Wed> Why Cantor's correspondence proof is mostly nonsense

---

In:

Roman Jan Duda, *The Origins of the Concept of Dimension*, in *Colloquium Mathematicum*, Instytut Matematyczny Polskiej Akademii Nauk (PAN), 1979, pp. 95–110. <https://bibliotekauki.pl/articles/727050.pdf>

... Duda notes at the bottom of page 96 (second page) that when Cantor discovered his method for mapping every point of a line into every point of a surface, he proclaimed:

« Je le vois, mais je ne le crois pas. » (“I see it, but I don't believe it.”)

Well, yes, he could not believe it because it's not true. As with points, the problem goes back to Euclid's definitions, which Duda also quotes on page 95 (first page):

1. A point is what has no part.
2. A line is what has length but not width.
3. The extremities of a line are points.
- ...
5. A surface is what has length and width only.

The problem is that in the more self-consistent (says Terry) two-fluid model, the phrases “no part” and “no width” are only limits one can approach, but never reach, with the atomic (smallest approximation used) units of the highest space necessarily becoming the finite components of the lower-dimensional units. Or, more bluntly, both surfaces (e.g., the film of a bubble) and lines (a fluid filament for a viscous fluid) consist of *at least 2D* atoms, and 3D in the case of actual atoms. The full correspondence between them breaks down because you can no longer expand the number of atoms in the filament case beyond that of a similar “filament-like” subset of the film.

If you try to solve this by making the atoms smaller, the ratio between film and filament gets *worse*, not better. For example, approximating with a 2×2 (4-atom) surface gives filaments with 2 atoms and a film-to-filament ratio of 2:1. In contrast, using atoms (approximating) 50 times smaller gives a 100×100 (10,000-atom) surface and 100-atom filaments for a film-to-filament ratio of 100:1.

Where Cantor screwed up was accidentally skipping over the scaling issue by *not* using a downward approximation. Had he done so, he might have noticed that he was packing points on the line infinitely more densely than on the surface. You can declare that “okay” if you want, of course, but he should have, at the very least, explicitly called out the difference in number densities. Being more realistic about number densities makes his finding pointless, since he's getting his “I don't believe it” result by packing an infinite number of infinitely finer-grained filament subsets of the surface, eventually approximating points, into the unfairly down-scaled line.

Cantor proved nothing except his sloppiness in scaling. [2026-01-07.13:18 EST Wed]