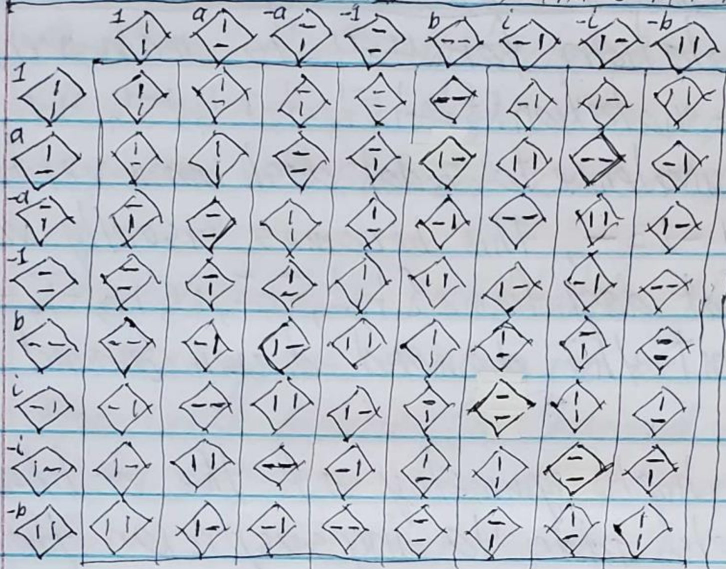
2015-11-09. 22:45 Mon

2015-11-10. 21:49

Dash, character, algebraic, and octahedral 4x4 products



2015-11-11. 10:51 Wed addendum part 2

Remarkably, the ccw octahedral face products (triple products) of the 4x4 non-real axes ⟨i,-i⟩, ⟨a,-a⟩, ⟨b,-b⟩ always give -1, regardless of starting point. Eg. (-a)(b)(i) = (b)(i)(-a) = (i)(-a)(b) = -1. Similarly, the cw (clockwise) face products all give +1. I did not expect this symmetry, identical to the octahedrons, to emerge with a mix of $\sqrt{1}$ and $\sqrt{-1}$.

10:59 end addendum Part 2

2015-11-10. 20:09 Tue

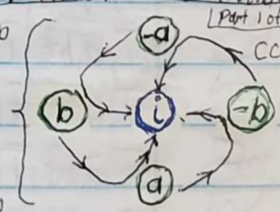
	1	a	-a	-1	b	i	-i	-b
1	1	a	-a	-1	b	i	-i	-b
a	a	1	-1	-a	-i	-b	b	i
-a	-a	-1	1	a	i	b	-b	-i
-1	-1	-a	a	1	-b	-i	i	b
b	b	i	-i	-b	1	a	-a	-1
i	i	b	-b	-i	-a	-1	1	a
-i	-i	-b	b	i	a	1	-1	-a
-b	-b	-i	i	b	-1	-a	a	1

$ab = -i$, $ba = i$
$ai = -b$, $ia = b$
$bi = a$, $ib = -a$
$a^2 = b^2 = 1$, $i^2 = -1$

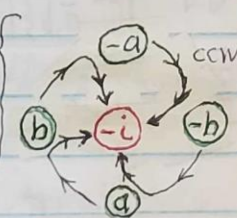
2015-11-11. 10:14 Wed addendum

Part 1 of 2

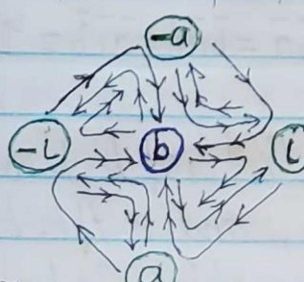
i-on-top top view of top half



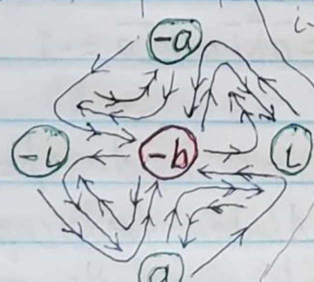
i-on-top top view of bottom half



10:50 addendum end (part 1)



[b on top] Top View of top half



[b on top] Top View of bottom half

2015-11-10. 21:46

Terry Bollinger 2015-11-10, 21:49 Tue

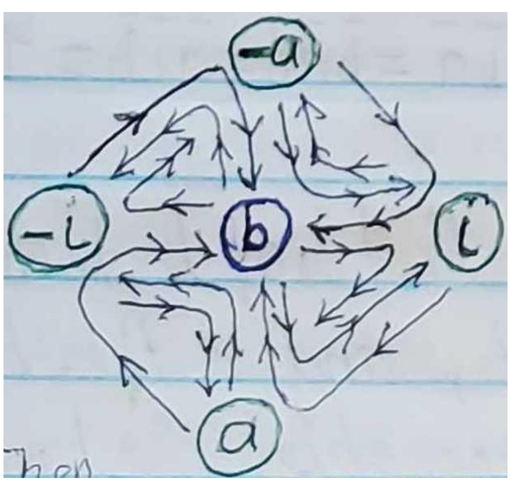
[2015-11-09.22:45 Mon> [2015-11-10.21:49]
[Dash, character, algebraic, and octahedral 4x4 products]

	^{1}⟨¦⟩	^{a}⟨⊥⟩	$^{-a}$⟨⊤⟩	$^{-1}$⟨=⟩	^{b}⟨--⟩	^{i}⟨-|⟩	$^{-i}$⟨|-⟩	$^{-b}$⟨||⟩
^{1}⟨¦⟩	^{1}⟨¦⟩	^{a}⟨⊥⟩	$^{-a}$⟨⊤⟩	$^{-1}$⟨=⟩	^{b}⟨--⟩	^{i}⟨-|⟩	$^{-i}$⟨|-⟩	$^{-b}$⟨||⟩
^{a}⟨⊥⟩	^{a}⟨⊥⟩	^{1}⟨¦⟩	$^{-1}$⟨=⟩	$^{-a}$⟨⊤⟩	$^{-i}$⟨|-⟩	$^{-b}$⟨||⟩	^{b}⟨--⟩	^{i}⟨-|⟩
$^{-a}$⟨⊤⟩	$^{-a}$⟨⊤⟩	$^{-1}$⟨=⟩	^{1}⟨¦⟩	^{a}⟨⊥⟩	^{i}⟨-|⟩	^{b}⟨--⟩	$^{-b}$⟨||⟩	$^{-i}$⟨|-⟩
$^{-1}$⟨=⟩	$^{-1}$⟨=⟩	$^{-a}$⟨⊤⟩	^{a}⟨⊥⟩	^{1}⟨¦⟩	$^{-b}$⟨||⟩	$^{-i}$⟨|-⟩	^{i}⟨-|⟩	^{b}⟨--⟩
^{b}⟨--⟩	^{b}⟨--⟩	$^{-i}$⟨-|⟩	$^{-i}$⟨|-⟩	$^{-b}$⟨||⟩	^{1}⟨¦⟩	^{a}⟨⊥⟩	$^{-a}$⟨⊤⟩	$^{-1}$⟨=⟩
^{i}⟨-|⟩	^{i}⟨-|⟩	^{b}⟨--⟩	$^{-b}$⟨||⟩	$^{-i}$⟨|-⟩	$^{-a}$⟨⊤⟩	$^{-1}$⟨=⟩	^{1}⟨¦⟩	^{a}⟨⊥⟩
$^{-i}$⟨|-⟩	$^{-i}$⟨|-⟩	$^{-b}$⟨||⟩	^{b}⟨--⟩	^{i}⟨-|⟩	^{a}⟨⊥⟩	^{1}⟨¦⟩	$^{-1}$⟨=⟩	$^{-a}$⟨⊤⟩
$^{-b}$⟨||⟩	$^{-b}$⟨||⟩	$^{-i}$⟨|-⟩	^{i}⟨-|⟩	^{b}⟨--⟩	$^{-1}$⟨=⟩	$^{-a}$⟨⊤⟩	^{a}⟨⊥⟩	^{1}⟨¦⟩

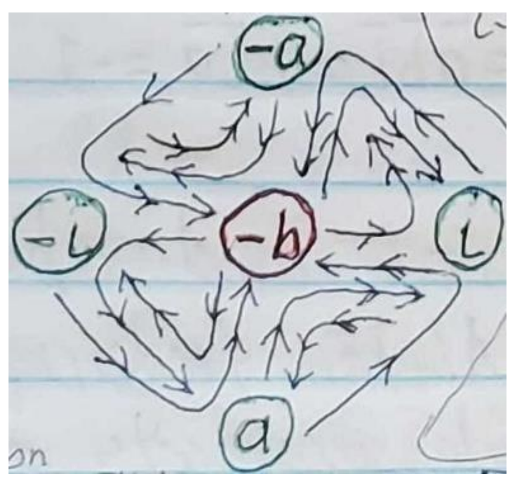
[2015-11-10.20:09 Tue>

	1	a	$-a$	-1	b	i	$-i$	$-b$
1	1	a	$-a$	-1	b	i	$-i$	$-b$
a	a	1	-1	$-a$	$-i$	$-b$	b	i
$-a$	$-a$	-1	1	a	i	b	$-b$	$-i$
-1	-1	$-a$	a	1	$-b$	$-i$	i	b
b	b	i	$-i$	$-b$	1	a	$-a$	-1
i	i	b	$-b$	$-i$	$-a$	-1	1	a
$-i$	$-i$	$-b$	b	i	a	1	-1	$-a$
$-b$	$-b$	$-i$	i	b	-1	$-a$	a	1

$ab = -1,$	$ba = i$
$ai = -b,$	$ia = b$
$bi = a,$	$ib = -a$
$a^2 = b^2 = 1,$	$i^2 = -1$



[b on top (of the octahedron)]
Top View of top half

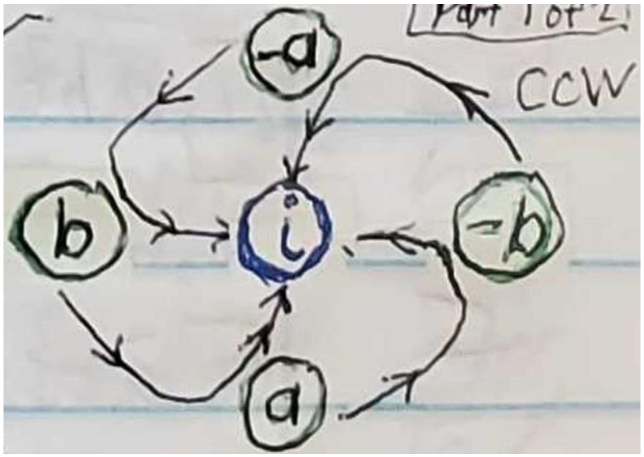


[b on top (but removed here)]
Top View of bottom half

[2015-11-10.21:46]
Terry Bollinger 2015-11-10.21:49 Tue

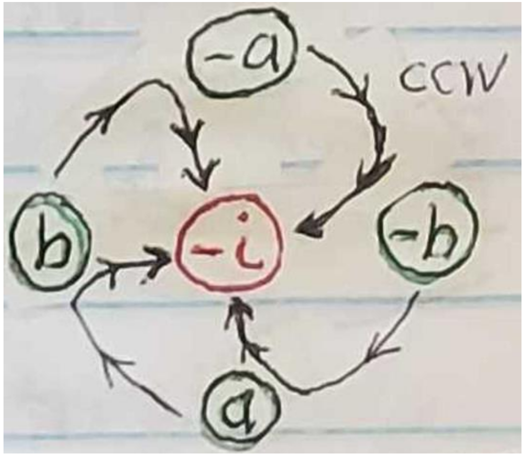
[Note: Addendums 1 and 2 are on the page 3.]

[2015-11-11.10:14 Wed addendum> [Part 1 of 2]



i-on-top
top view of
top half

[Note: These are the top five vertices of an octahedron, with the bottom (-i) vertex hidden behind (i). The curved paths show the CCW multiplications of one face's vertices. (This note added 2023-12-16.08:52 Sat)]



i-on-top
top view of
bottom half

[Note: Face rotation is still CCW if viewed from outside the octahedron. However, it looks CW here due to removing the top half of the octahedron, giving an inside-out view. (This note added 2023-12-16.08:53 Sat)]

[2025-11-11.10:50 addendum end (part 1)]

[2015-11-11.10:51 Wed addendum part 2>

Remarkably, the ccw octahedral face products (triple products) of the 4x4 non-real axes <i,-i>, <a,-a>, <b,-b> always give -1, regardless of starting point. E.g. (-a)(b)(i)=(b)(i)(-a)=(i)(-a)(b)=-1. Similarly, the cw (clockwise) face products all give +1. I did not expect this symmetry, identical to the octahedrons, to emerge with a mix of √[1] and √[-1].

[2025-11-11.10:59 end addendum Part 2]