Implications of a Math Error on Page 420 of Einstein's 1907 Special Relativity Summary

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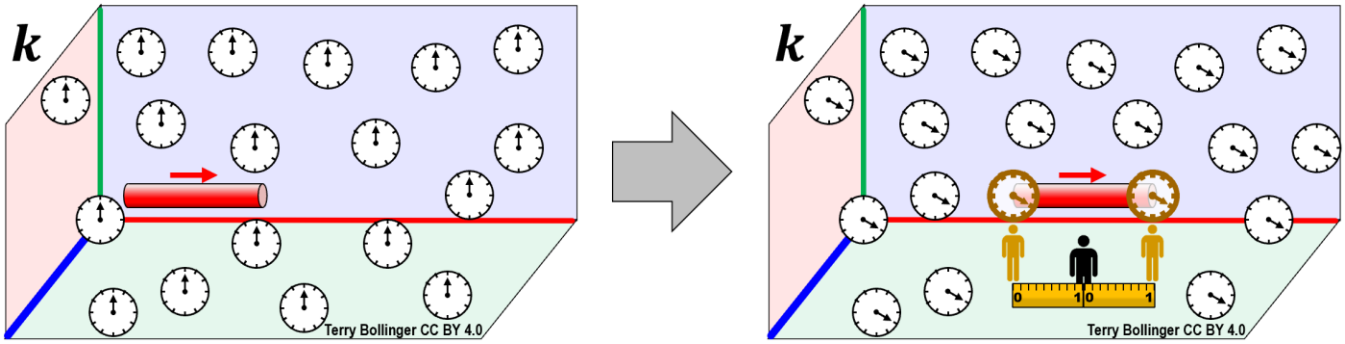
2024-09-23.23:30 EDT Mon



Einstein first identified the twin paradox as a continuous slowdown in clouds of clocks synchronized clocks.

On September 22, 2024, Sabine Hossenfelder posted a YouTube video analysis [1] of a February 2024 paper by Ilan Meltzer and Yoav Sagion [2] on how current technology may be sufficient to implement a quantum version of Einstein's twin paradox. Sina Loriani *et al.* first proposed the quantum twin concept in a 2019 paper [3], replacing independent twins with a superposition of two states of a single atom traveling along different gravitational paths. The quantum twin approach then uses high-sensitivity matter-wave interferometry to assess the resulting age differences accumulated along the two paths. Depending on the setup, this approach allows the comparison of paths that differ based on special relativity, gravity, or a combination of the two.

The impact of acceleration forces is relevant in such experiments due to the equivalence principle, based on the following hypothesis: If the acceleration forces encountered during the launch and turnaround of a spaceship are critical to creating the asymmetric time dilations seen in the twin problem, then gravity may also impact the twin problem through the Einstein equivalence of gravitational forces to acceleration forces. In her video, Hossenfelder describes the importance of acceleration at timestamp 1:35 [4]:

"A change of direction is an acceleration ... The twins can't meet again until one turns around ... The one who turns around [is] accelerated. Hence, his time runs slower. So, no paradox."

The best resolution of how acceleration and acceleration forces play into the twin paradox is to examine exactly how Einstein first derived the twin paradox and how his method dealt with acceleration and acceleration forces. The author first discussed this issue in a comment on the Hossenfelder video on both YouTube and Patreon [5], and the rest of this paper repeats that analysis. However, this analysis cannot ignore a critical related point in the early Einstein papers: a mathematical error he made deriving his famous fully symmetric transformation equations between inertial frames. Einstein appears to have had concerns that his equations were too simple since he explicitly qualifies their correctness in a footnote assumption that is not valid.

In his 1911 paper *The Theory of Relativity* [6] (German: *Die Relativitäts-Theorie* [7]), he predicted experimentally asymmetric time dilation by arguing that if you fill a large space with a cloud of clocks that are at rest relative to the home twin's clock and synchronized with that home clock, then at every point in the journey when the moving twin his clock touches one of the home-synchronized clocks, it will show a time delay directly proportional to how far the clock has traveled inertially.

While Einstein includes a direction reversal for the traveling clock, he adds this point only *after* making his continuously accumulated delay argument. He reverses the direction of travel not to create a time reversal but to show that the accumulation of time delay in the traveling clock is more than some kind of illusion created by the space separation of the clocks. Bringing the traveling clock home shows that the same time dilations recorded continuously by distant clocks also apply when the two clocks (or twins) are again in immediate contact. Unfortunately, Einstein's inclusion of a direction reversal left many with the incorrect impression that this was his explanation of why the asymmetry occurs. However, that was never in Einstein's original argument, which focused on continual delay.

Einstein's 1911 paper originated as an attempt to transcribe a lecture he gave on January 16. Judging by references in the text, Einstein must have drawn several blackboard figures to illustrate arguments, but the note-taker only had time to capture the first and least important of these figures. At least one important bridge sentence is missing from the transcript and resulting paper. Finally, the unusual (this was Einstein) and mostly non-mathematical logic of his lecture likely never made it easy to follow, even with the figures and a better transcript. Such factors likely made the paper difficult to translate well and may explain why the paper is not widely available to English readers. While verifying my translation of the German article as best I could, I also created an expanded translation in which I attempted to recreate some of the missing figures [8]. I also added an appendix from Einstein's 1907 paper on translating xyzt coordinates between inertial frames.

Regarding the gravity case, the simplest and most algorithmically reliable way to recognize when continuous asymmetric time dilation applies is to ask which system receives new energy in the form of linear momentum. These linear momentum excitations, which are more like angular momentum excitations than one might expect, involve energy transfers. Thus, they are not relativistic — the energy exchange looks the same to all viewers in all inertial frames.

Importantly but subtly, accumulating linear momentum energy is not always associated with feeling acceleration forces. Time dilation occurs only in systems that accumulate linear momentum energy. Thus, the twin in the rocket, energized by the addition of rocket-derived linear momentum, undergoes time dilation relative to a background of home clocks that receive no such linear momentum excitation energy. In sharp contrast, gravitational acceleration forces on a planetary surface *do not* add linear momentum energy because they induce no motion in the objects. Ironically, the only case where gravity continually adds linear momentum energy, and thus time dilation, is when the object *stops* feeling acceleration and goes into freefall. Thus, again, feeling acceleration forces and successfully adding linear momentum energy are identical, and only the latter causes asymmetric time dilation.

The common but completely incorrect belief that the home twin and traveling twin observe the same Lorentz length contraction and time dilation effects when examining each other's systems stems from a math error that Einstein made on page 420 of his 1907 summary of conclusions he had drawn from his special relativity paper [9]. Einstein was aware of the possible problem since he flagged the possibility in a footnote:

"1) This conclusion [that the transformation equations are fully symmetric from both frame perspectives] is based on the physical assumption that the length of a ruler and the speed of a clock do not suffer any permanent change as a result of these objects being set in motion and brought to rest again." — A. Einstein, 1907

Einstein's assumption is false for the abovementioned reason: Only one system receives a linear momentum energy excitation. Furthermore, a careful examination of the implications of that energy addition shows that Lorentz contraction is *not* the abstract, math-only operation that Einstein assumed but an asymmetric, energy-consuming physical contraction that occurs only in the system that undergoes linear momentum excitation. The physical consequences of not performing this physical contraction are best known as Bell's ship paradox.

What happens experimentally is that only the (generally much larger) unaccelerated or "launch" system observes fully real and unequivocally physical contraction of the moving system and continuous slowing of its clocks. Ask

any particle accelerator operator for examples. What the moving system observes is far more complicated but was first noticed in quasar jets. The moving twin observes relativistic Doppler time *speedup* and length contraction in the forward direction. Looking backward, however, the moving twin observes a relativistic Doppler *slowdown* with length *expansion*. The forward view of faster time is the most important since it becomes reality as the ship reaches objects in that direction. The Lorentz factor interpretation applies only to the equatorial view of the ship, which in turn boils down to saying that outside particles passing through the ship appear Lorentz contracted and time dilated only for their brief period of traversal. While brief, this transition period is vital because it enables full Poincaré symmetry of the two frames at the physics level.

References

[1] S. Hossenfelder, *This Could Test the Quantum Twin Paradox*, Sabine Hossenfelder (YouTube), Sep 22, 2024. https://youtu.be/uk4m3Xe6b7E.

[2] I. Meltzer and Y. Sagi, *Atomic clock interferometry using optical tweezers*, Physical Review A **110** (3), 32602 [Feb. 22] (2024). https://arxiv.org/abs/2402.14412.

[3] S. Loriani et al., *Interference of clocks: A quantum twin paradox*, Science Advances **5** (10), eaax8966 (2019). https://www.science.org/doi/full/10.1126/sciadv.aax8966.

[4] S. Hossenfelder, *This Could Test the Quantum Twin Paradox*, Sabine Hossenfelder (YouTube), Sep 22, 2024. https://youtu.be/uk4m3Xe6b7E&t=1m35s. The relevant quote begins at timestamp 1:35.

[5] Comment by Terry Bollinger on 2024-09-22.22:34 EDT on '*This Could Test the Quantum Twin Paradox*, Sabine Hossenfelder (YouTube) Sep 22, 2024.' https://youtu.be/uk4m3Xe6b7E&lc=Ugw3Ff22PBphmk1bsrR4AaABAg, Patreon: https://www.patreon.com/posts/112386827.

[6] A. Einstein, *The Theory of Relativity*, Naturforschende Gesellschaft, Zürich, Vierteljahresschrift **56**, 1–14 [Jan. 16] (1911). https://sarxiv.org/ref.1911-01-16.germ.pdf. Translation by Terry Bollinger.

[7] A. Einstein, *Die Relativitäts-Theorie*, Naturforschende Gesellschaft, Zürich, Vierteljahresschrift **56**, 1–14 [Jan. 16] (1911). https://sarxiv.org/ref.1911-01-16.germ.pdf. Exact digital recreation by Terry Bollinger of German journal paper.

[8] A. Einstein, *The Theory of Relativity [with Figures]*, Naturforschende Gesellschaft, Zürich, Vierteljahresschrift **56**, (1–14) [Jan. 16] (1911). https://sarxiv.org/ref.1911-01-16.figs.pdf. Translation by Terry Bollinger with attempts to recreate implied figures.

[9] A. Einstein, *Coordinate-Time Transformation* [Sec. 3, pp. 418–420, in '*About the Principle of Relativity and the Conclusions Drawn from It*,' pp. 411–462], Jahrbuch der Radioaktivität und Elektronik **4** (4), 418–420 (1907). https://sarxiv.org/ref.1907-04-04.0418.engl.pdf.