The Missing Momentum Mass Puzzle

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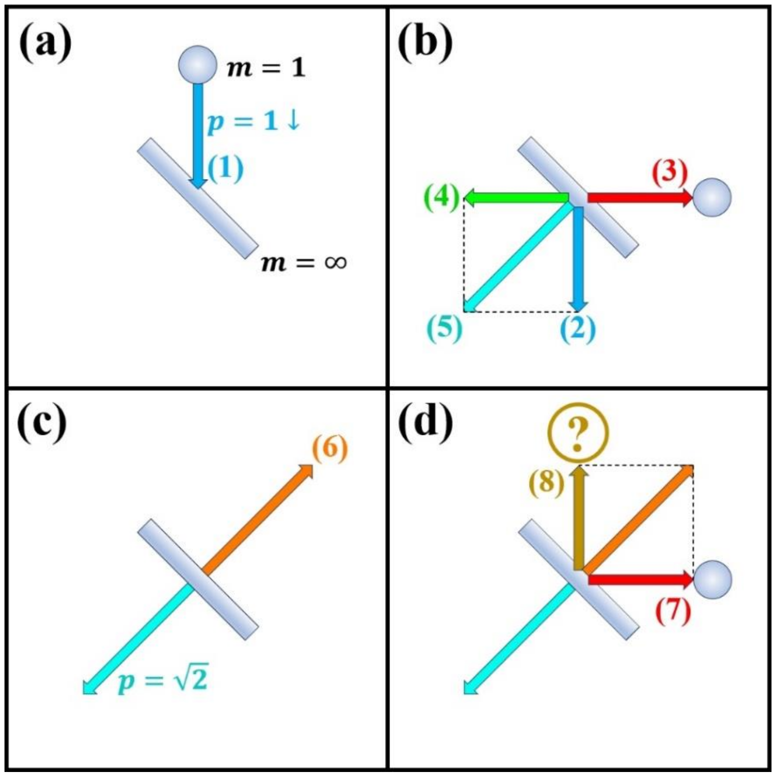

Figure 1. *The Mystery of the Missing Momentum Mass*

While watching a nice YouTube video on Feynman's Sprinkler problem and trying to reduce the problem down to particle interactions, I inadvertently came across a simple momentum problem — not much different from a game of pool, really — that nonetheless left me bewildered for hours (Fig. 1). There's a solution. However, I'm curious whether I'm the only one who had trouble seeing it.

The first point to remember is one of the odder features of momentum: For any given amount of energy, you can, in principle, create an indefinitely large amount of momentum [1]. The restriction is that the momentum created must be self-canceling — that is, it must consist of action-reaction pairs of masses traveling in *opposite* directions. Increasing the total momentum magnitude is important for understanding Fig. 1 since, in that figure, one momentum vector ends up spawning several more momentum vectors. That's not the mystery — that's just how momentum works.

Figure 1(a) shows the setup. You have a ball with one unit of mass in a gravity-free environment — in space or, more mundanely, a pool table — and send it towards a very heavy plate or bar at a 45° angle to the ball's path. You choose a velocity so that, conveniently, the downward (blue for bottom) momentum p also works out to be 1.

The moment the ball hits the plate, two things happen. Since the bar fully stops all downward momentum of the ball — the ball moves sideways after that — The first step **(1)** is that all of the blue-for-bottom momentum transfers into the bar **(2)**. You see this same kind of full-forward-stop momentum transfer between balls in Newton's cradle when each ball is blocked by the next one.

The next two steps invoke the principle I mentioned: you can create an indefinitely large number of momentum *pairs* — Newton's action-reaction pairs — if you set up your system correctly. Even though *all* of the original momentum transfers into the bar, the rebound effect of the bar creates *two more* momentum units, each equal in magnitude to the original momentum unit. You cannot do that with energy, but you can do it repeatedly with momentum! Each pair production has a small energy cost, but that cost is so incredibly small that it is *almost* immeasurable, light sails in spacecraft being the exception.

The first red-for-right momentum unit **(3)** accompanies the reflected ball, while the paired and canceling lime-for-left unit goes into the heavy bar **(4)**. Generating this pair of momentum vectors is a beautiful example of the canceling rule for momentum: You can double the total magnitude of the momentum easily, but the two created units *must* be mutually canceling to conserve momentum overall in the system.

The bar has now received not one but *two* units of momentum. However, these units add like vectors, not simple numbers, so in Fig. 5(b), the blue-bottom and lime-left momentum units add up to the diagonal of a square. This final lime-blue (cyan) momentum vector sum **(5)** fully characterizes the new momentum state of the bar. While it may seem odd that the bar now has *more* momentum than the ball that hit it, that's how momentum works, specifically through momentum pair creation. Spacecraft use this momentum enhancement effect in perpendicular sun sails to *double* the momentum each sun photon imparts to the spacecraft.

As always, however, *total* momentum must be conserved! The bar was, at first, fully at rest, so the cyan vector created by the collision *must* have a canceling vector somewhere away from the bar. Fig. 1(c) shows this new canceling momentum vector as a brown vector **(6)**.

The next question is this: What is the physical mass or masses in which this brown momentum vector resides? Except for photons — not part of this objects-only problem — momentum can reside only in objects with mass.

Fig. 1(d) shows part of the answer: The red-right unit of momentum **(7)** travels with the ball that bounced. Subtracting (vector subtraction) of this known momentum carrier from the brown vector then gives the remaining momentum vector needed to make everything work out correctly: A tan-for-top unit vector **(8)** that, when added to the red-right vector, gives the orange vector that keeps everything balanced. Everything works!

Except for one little problem: *There is no mass to hold the tan-for-top momentum vector in the figure*.

The question, then, is this: Where is the object that holds the tan-for-top momentum?

The answer may be obvious for some of you — perhaps most of you. I was not so fortunate and had difficulty seeing the answer for some time. Even when I did, I'm not *entirely* sure the answer is as simple as it seems.

What do you say?

References

[1] T. Bollinger, *How to Convert One Green Photon Into Two Locomotives of Momentum*, TAO Physics **2021**, 0930 [Nov. 30] (2021). DOI: doi.org/10.48034/20210930