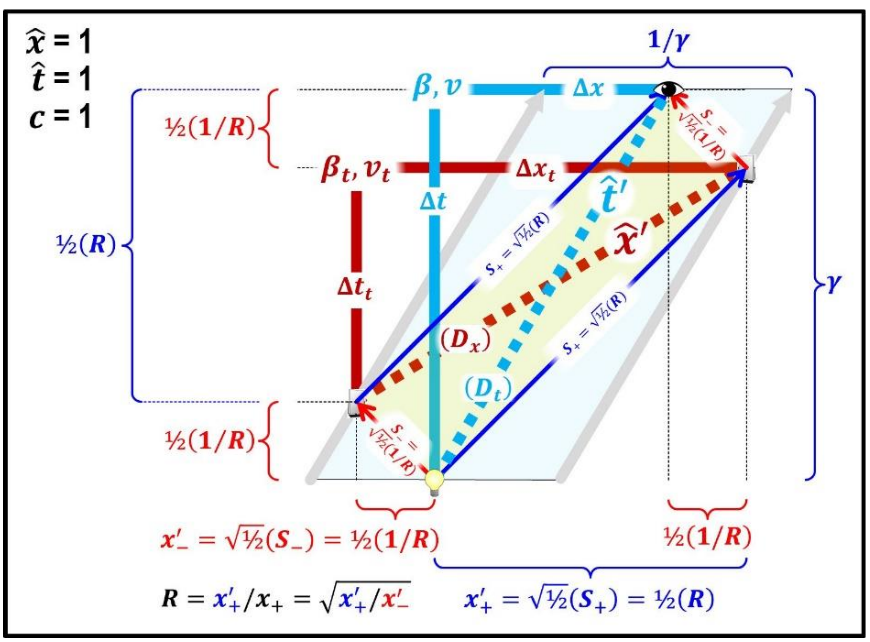
Special Relativity Conversions with Relativistic Doppler, Binary Rapidity, and Age Gradients

	Velocity $v =$	Unitless Velocity $\beta =$	Forward Light Paths Ratio (Relativistic Doppler Factor) $R =$	Rapidity $w =$	Binary Rapidity $\rho =$	Lorentz Factor $\gamma =$	Diagonal Factor $D =$	Age Gradient $\alpha =$	Traveler's Gradient $\alpha_+ =$	In-Frame Gradient $\alpha' =$
Best →	v	$\frac{v}{c}$	$\sqrt{\frac{1+\beta}{1-\beta}}$	$\ln R$	$\log_2 R$	$\frac{R+R^{-1}}{2}$	$\sqrt{2\gamma^2-1}$	$-\frac{\beta\gamma}{c}$	$-\alpha$	$-\frac{v}{c^2}$
Given ↓ v	v	$\frac{v}{c}$	$\sqrt{\frac{c+v}{c-v}}$	$\ln\sqrt{\frac{c+v}{c-v}}$	$\log_2\sqrt{\frac{c+v}{c-v}}$	$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$	$\sqrt{\frac{c^2+v^2}{c^2-v^2}}$	$-\frac{v}{c\sqrt{c^2-v^2}}$	$\frac{v}{c\sqrt{c^2-v^2}}$	$-\frac{v}{c^2}$
β	$c\beta$	β	$\sqrt{\frac{1+\beta}{1-\beta}}$	$\ln\sqrt{\frac{1+\beta}{1-\beta}}$	$\log_2\sqrt{\frac{1+\beta}{1-\beta}}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\sqrt{\frac{1+\beta^2}{1-\beta^2}}$	$-\frac{\beta}{c\sqrt{1-\beta^2}}$	$\frac{\beta}{c\sqrt{1-\beta^2}}$	$-\frac{\beta}{c}$
R	$c\frac{R-R^{-1}}{R+R^{-1}}$	$\frac{R-R^{-1}}{R+R^{-1}}$	R	$\ln R$	$\log_2 R$	$\frac{R+R^{-1}}{2}$	$\sqrt{\frac{R^2+R^{-2}}{2}}$	$-\frac{R-R^{-1}}{2c}$	$\frac{R-R^{-1}}{2c}$	$-\frac{R-R^{-1}}{2c}$
w	$c\frac{e^w-e^{-w}}{e^w+e^{-w}}$	$\frac{e^w-e^{-w}}{e^w+e^{-w}}$	e^w	w	$\frac{w}{\ln 2}$	$\frac{e^w+e^{-w}}{2}$	$\sqrt{\frac{e^{2w}+e^{-2w}}{2}}$	$-\frac{e^w-e^{-w}}{2c}$	$\frac{e^w-e^{-w}}{2c}$	$-\frac{e^w-e^{-w}}{2c}$
ρ	$c\frac{2^\rho-2^{-\rho}}{2^\rho+2^{-\rho}}$	$\frac{2^\rho-2^{-\rho}}{2^\rho+2^{-\rho}}$	2^ρ	$\rho\ln 2$	ρ	$\frac{2^\rho+2^{-\rho}}{2}$	$\sqrt{\frac{2^{2\rho}+2^{-2\rho}}{2}}$	$-\frac{2^\rho-2^{-\rho}}{2c}$	$\frac{2^\rho-2^{-\rho}}{2c}$	$-\frac{2^\rho-2^{-\rho}}{2c}$
γ	$c\sqrt{1-\frac{1}{\gamma^2}}$	$\sqrt{1-\frac{1}{\gamma^2}}$	$\sqrt{\frac{\gamma+\sqrt{\gamma^2-1}}{\gamma-\sqrt{\gamma^2-1}}}$	$\ln\sqrt{\frac{\gamma+\sqrt{\gamma^2-1}}{\gamma-\sqrt{\gamma^2-1}}}$	$\log_2\sqrt{\frac{\gamma+\sqrt{\gamma^2-1}}{\gamma-\sqrt{\gamma^2-1}}}$	γ	$\sqrt{2\gamma^2-1}$	$-\frac{\sqrt{\gamma^2-1}}{c}$	$\frac{\sqrt{\gamma^2-1}}{c}$	$-\frac{\sqrt{\gamma^2-1}}{c}$
D	$c\sqrt{\frac{D^2-1}{D^2+1}}$	$\sqrt{\frac{D^2-1}{D^2+1}}$	$\sqrt{\frac{1+\sqrt{\frac{D^2-1}{D^2+1}}}{1-\sqrt{\frac{D^2-1}{D^2+1}}}}$	$\ln\sqrt{\frac{1+\sqrt{\frac{D^2-1}{D^2+1}}}{1-\sqrt{\frac{D^2-1}{D^2+1}}}}$	$\log_2\sqrt{\frac{1+\sqrt{\frac{D^2-1}{D^2+1}}}{1-\sqrt{\frac{D^2-1}{D^2+1}}}}$	$\sqrt{\frac{D^2+1}{2}}$	D	$-\sqrt{\frac{D^2-1}{2c^2}}$	$\sqrt{\frac{D^2-1}{2c^2}}$	$-\sqrt{\frac{D^2-1}{2c^2}}$
α	$-\frac{\alpha c^2}{\sqrt{1+\alpha^2c^2}}$	$-\frac{\alpha c}{\sqrt{1+\alpha^2c^2}}$	$\sqrt{\frac{\sqrt{1+\frac{1}{\alpha^2c^2}}+1}{\sqrt{1+\frac{1}{\alpha^2c^2}}-1}}$	$\ln\sqrt{\frac{\sqrt{1+\frac{1}{\alpha^2c^2}}+1}{\sqrt{1+\frac{1}{\alpha^2c^2}}-1}}$	$\log_2\sqrt{\frac{\sqrt{1+\frac{1}{\alpha^2c^2}}+1}{\sqrt{1+\frac{1}{\alpha^2c^2}}-1}}$	$\sqrt{1+\alpha^2c^2}$	$\sqrt{1+2\alpha^2c^2}$	α	$-\alpha$	$\frac{\alpha}{\sqrt{1+\alpha^2c^2}}$
α_+	$\frac{\alpha_+c^2}{\sqrt{1+\alpha_+^2c^2}}$	$\frac{\alpha_+c}{\sqrt{1+\alpha_+^2c^2}}$	$\sqrt{\frac{\sqrt{1+\frac{1}{\alpha_+^2c^2}}+1}{\sqrt{1+\frac{1}{\alpha_+^2c^2}}-1}}$	$\ln\sqrt{\frac{\sqrt{1+\frac{1}{\alpha_+^2c^2}}+1}{\sqrt{1+\frac{1}{\alpha_+^2c^2}}-1}}$	$\log_2\sqrt{\frac{\sqrt{1+\frac{1}{\alpha_+^2c^2}}+1}{\sqrt{1+\frac{1}{\alpha_+^2c^2}}-1}}$	$\sqrt{1+\alpha_+^2c^2}$	$\sqrt{1+2\alpha_+^2c^2}$	$-\alpha_+$	α_+	$-\frac{\alpha_+}{\sqrt{1+\alpha_+^2c^2}}$
α'	$-\alpha'c^2$	$-\alpha'c$	$\sqrt{\frac{1-\alpha'c}{1+\alpha'c}}$	$\ln\sqrt{\frac{1-\alpha'c}{1+\alpha'c}}$	$\log_2\sqrt{\frac{1-\alpha'c}{1+\alpha'c}}$	$\frac{1}{\sqrt{1+\alpha'^2c^2}}$	$\sqrt{\frac{1+\alpha'^2c^2}{1-\alpha'^2c^2}}$	$\frac{\alpha'}{\sqrt{1-\alpha'^2}}$	$-\frac{\alpha'}{\sqrt{1-\alpha'^2}}$	α'



All special relativity velocity factors are derivable using a light clock with two mirrors equidistant from a light-pulse emitter/detector. The distance between mirrors defines one length unit $\hat{x}$, and the time from pulse emission to return defines one time unit $\hat{t}$. Motion of the system along the axis of mirror separation requires *physical* repositioning (Bell's ship paradox) of the parts in the observer frame to ensure synchronous pulse returns. The new lengths then define the various velocity factors.

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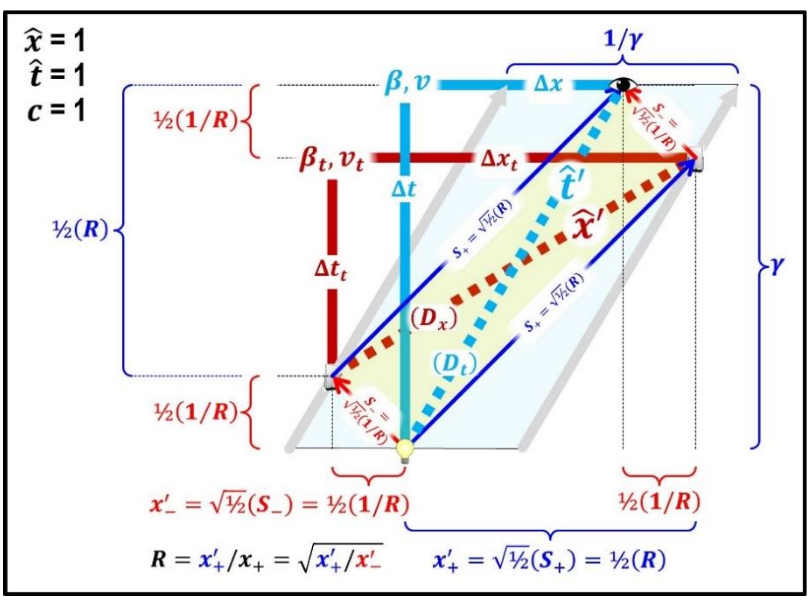
	Velocity $\boldsymbol{v}$ V =	*Unitless Velocity* $\boldsymbol{\beta}$ B =	*Forward Light Paths Ratio (Relativistic Doppler Factor)* $\boldsymbol{R}$ R =	*Rapidity* $\boldsymbol{w}$ W =	*Binary Rapidity* $\boldsymbol{\rho}$ P =	*Lorentz Factor* $\boldsymbol{\gamma}$ Y =	*Diagonal Factor* $\boldsymbol{D}$ D =	*Age Gradient* $\boldsymbol{\alpha}$ A =	*Traveler's Gradient* $\boldsymbol{\alpha_+}$ F =	*In-Frame Gradient* $\boldsymbol{\alpha'}$ J =
Best→	(V)	(V)/c	√((1+(B))/(1-(B)))	ln((R))	log_2((R))	((R)+(R)^(-1))/2	√(2((Y)^2)–1)	-(B)(Y)/c	-(A)	-(V)/(c^2)
Given↓										
$\boldsymbol{v}$: V	(V)	(V)/c	√((c+(V))/(c-(V)))	ln(√((c+(V))/(c-(V))))	log_2(√((c+(V))/(c-(V))))	1/√(1-((V)^2)/(c^2))	√(((c^2)+((V)^2))/((c^2)-((V)^2)))	-(V)/(c√((c^2)-((V)^2)))	(V)/(c√((c^2)-((V)^2)))	-(V)/(c^2)
$\boldsymbol{\beta}$: B	c(B)	(B)	√((1+(B))/(1-(B)))	ln(√((1+(B))/(1-(B))))	log_2(√((1+(B))/(1-(B))))	1/√(1-(B)^2)	√((1+(B)^2)/(1-(B)^2))	-(B)/(c√(1-(B)^2))	(B)/(c√(1-(B)^2))	-(B)/c
$\boldsymbol{D}$: R	c((R)-(R)^(-1))/((R)+(R)^(-1))	((R)-(R)^(-1))/((R)+(R)^(-1))	(R)	ln((R))	log_2((R))	((R)+(R)^(-1))/2	√(((R)^2+(R)^(-2))/2)	-((R)-(R)^(-1))/2c	((R)-(R)^(-1))/2c	-((R)-(R)^(-1))/2c
$\boldsymbol{w}$: W	c((e^(W))-e^(-(W)))/((e^(W))+e^(-(W)))	((e^(W))-e^(-(W)))/((e^(W))+e^(-(W)))	(e^(W))	(W)	(W)/ln(2)	((e^(W))+e^(-(W)))/2	√(((e^(2(W)))+(e^(-2(W))))/2)	-((e^(W))-e^(-(W)))/2c	((e^(W))-e^(-(W)))/2c	-((e^(W))-e^(-(W)))/2c
$\boldsymbol{\rho}$: P	c((2^(P))-(2^(-(P))))/((2^(P))+(2^(-(P))))	((2^(P))-(2^(-(P))))/((2^(P))+(2^(-(P))))	(2^(P))	(P)ln(2)	(P)	((2^(P))+(2^(-(P))))/2	√(((2^(2(P)))+(2^(-2(P))))/2)	-((2^(P))-(2^(-(P))))/2c	((2^(P))-(2^(-(P))))/2c	-((2^(P))-(2^(-(P))))/2c
$\boldsymbol{\gamma}$: Y	c√(1-1/((Y)^2))	√(1-1/((Y)^2))	√(((Y)+√(((Y)^2)-1))/((Y)-√(((Y)^2)-1)))	ln(√(((Y)+√(((Y)^2)–1))/((Y)-√(((Y)^2)-1))))	log_2(√(((Y)+√(((Y)^2)-1))/((Y)-√(((Y)^2)-1))))	(Y)	√(2((Y)^2)–1)	-√(((Y)^2)-1)/c	√(((Y)^2)-1)/c	-√(((Y)^2)-1)/c
$\boldsymbol{D}$: D	c√((((D)^2)-1)/(((D)^2)+1))	√((((D)^2)-1)/(((D)^2)+1))	√((1+√((((D)^2)-1)/(((D)^2)+1)))/(1-√((((D)^2)-1)/(((D)^2)+1))))	ln(√((1+√((((D)^2)-1)/(((D)^2)+1)))/(1-√((((D)^2)-1)/(((D)^2)+1)))))	log_2(√((1+√((((D)^2)-1)/(((D)^2)+1)))/(1-√((((D)^2)-1)/(((D)^2)+1)))))	√((((D)^2)+1)/2)	(D)	-√((((D)^2)-1)/(2(c^2)))	√((((D)^2)-1)/(2(c^2)))	-√((((D)^2)-1)/(2(c^2)))
$\boldsymbol{\alpha}$: A	-((A)(c^2))/√(1+((A)^2)(c^2))	-(A)c/√(1+((A)^2)(c^2))	√((√(1+1/(((A)^2)(c^2)))+1)/(√(1+1/(((A)^2)(c^2)))-1))	ln(√((√(1+1/(((A)^2)(c^2)))+1)/(√(1+1/(((A)^2)(c^2)))-1)))	log_2(√((√(1+1/(((A)^2)(c^2)))+1)/(√(1+1/(((A)^2)(c^2)))-1)))	√(1+((A)^2)(c^2))	√(1+2((A)^2)(c^2))	(A)	-(A)	(A)/√(1+((A)^2)(c^2))
$\boldsymbol{\alpha_+}$: F	((F)(c^2))/√(1+((F)^2)(c^2))	((F)c)/√(1+((F)^2)(c^2))	√((√(1+1/(((F)^2)(c^2)))+1)/(√(1+1/(((F)^2)(c^2)))-1))	ln(√((√(1+1/(((F)^2)(c^2)))+1)/(√(1+1/(((F)^2)(c^2)))-1)))	log_2(√((√(1+1/(((F)^2)(c^2)))+1)/(√(1+1/(((F)^2)(c^2)))-1)))	√(1+((F)^2)(c^2))	√(1+2((F)^2)(c^2))	-(F)	(F)	-(F)/√(1+((F)^2)(c^2))
$\boldsymbol{\alpha'}$: J	-(J)(c^2)	-(J)c	√((1-(J)c)/(1+(J)c))	ln(√((1-(J)c)/(1+(J)c)))	log_2(√((1-(J)c)/(1+(J)c)))	1/√(1+(((J)^2)(c^2))	√((1+(((J)^2))(c^2))/(1-(((J)^2))(c^2)))	(J)/√(1-((J)^2))	-(J)/√(1-((J)^2))	(J)



The table above has text versions of equations that are compatible with Google's equation processor. After replacing the capital letters with numeric values, they can be pasted directly into a Google prompt to get numeric results.

Notes: The forward light path ratio R gives a geometric interpretation of the relativistic Doppler factor. Binary rapidity is easier to use in figures that standard rapidity. Age gradients quantify the degree of non-simultaneity along the length of objects, e.g., Einstein's trains.