Why Age Gradients are Useful in Special Relativity

Terry Bollinger
2022-09-21.00:00 EDT Wed *[Updated 2022-09-23] [Tweaked 2022-09-27]*

[Draft Paper]

Age Gradients as Time Zones

The clocks in a region or object with an *age gradient* no longer show one moment in time, but instead show a smoothly changing range of times as you move in a specific direction.

Time zones are human-built age gradients since they require clock settings to change as one moves a distance in space. Each clock provides a good definition of *local* physics in terms of sun position, but to an observer in an orbiting station, these conflicting and thus asynchronous definitions of time exist simultaneously.

Age gradients exist naturally in all moving objects, although they only become noticeable in objects that are very long or moving at relativistic speeds. All versions of relativistic age gradients behave like westbound planes flying over fine-grained versions of ordinary time zones. Thus just as the tail of an exceptionally long westbound plane might think it is already noon while the nose of the plane still sees morning, the trailing edge of a moving object sees a slightly later time than its leading edge.

There is nothing new or exotic in why moving objects have age gradients since the effect directly corresponds to the interchangeability of space and time in special relativity. A moving object acquires its own set of space and time coordinates, which means that any attempt to view that object from a rest frame slices across a mix of space and time coordinates in the moving object. Age gradients result from slicing across coordinates that are no longer purely space but include some time. The shortening of a fast-moving object along its length — its Lorentz compression — is a consequence of the same slicing. The age gradient of a moving object thus is just a description of how the time labels change across the same slice that defines the Lorentz compression of an object. The two effects cannot be separated, and both have testable consequences.

Many readers should now be asking the obvious question: If age gradients are as much a part of special relativity as Lorentz contraction, why have I never heard of them before?

That is an oddly difficult question to answer, perhaps falling into the same category as asking someone, "how do you ride a bike?" Experienced users of special relativity are so adept at switching points of view that they do not need age gradients. Instead, they switch to the moving object's point of view as needed, eliminating any need to deal with the odd and decidedly non-intuitive asymmetric dynamics created by age gradients.

Nonetheless, one of the founding principles of special relativity is that the dynamics of *any* frame can account for the physics of *all* frames. No matter how much we assert that we accept the power and completeness of special relativity, switching the point of view too readily undercuts that belief and potentially undermines our understanding of special relativity. The first step in staying in-frame is, not surprisingly, accepting age gradients.

The Universal Age Gradient

The core of every age gradient is the expression v/c^2 with units of seconds per meter. Since age gradients arise from a race between the velocity of the object and the velocity of light, this close connection to velocity delays shows that age gradients are merely a different way of representing standard relativistic dynamics. Depending on the observer and metrics used, several variants are possible. The *universal age gradient* α_u, for example, describes how the universe looks to a fast-moving traveler:

$$\boldsymbol{\alpha_u} = \gamma \frac{v}{c^2} = \frac{\boldsymbol{\gamma\beta}}{\boldsymbol{c}} = \frac{\boldsymbol{\beta}}{\boldsymbol{c\sqrt{1-\beta^2}}} = \frac{v}{c^2\sqrt{1-v^2/c^2}} = \frac{\boldsymbol{v}}{\boldsymbol{c\sqrt{c^2-v^2}}}$$

Gamma γ is the Lorentz factor of special relativity, and beta is the unitless expression of velocity as a fraction of lightspeed, $\beta = v/c$. When using the β version, expressing c in proportional units such as one lightyear per year or one light-nanosecond (roughly one foot) per nanosecond simplifies the calculation. For velocities close to c, $\beta \cong 1$ and the gamma-proportional pseudo universal age gradient $\boldsymbol{\alpha_{\sim u} = \gamma/c}$ can be used instead.

Age Gradients are Not Time Dilations

A critical point is this: Age gradients are independent of time dilation, and both contribute to any estimation of the final destination time. This separation is why observers in two frames can see the other as time dilated. If a spaceship requires 100,000 years to cross a galaxy at nearly the speed of light, then regardless of time dilation, destination clocks should show that the ship took 100,000 years to arrive. The age gradient captures this delay while ensuring that all galaxy clocks add time very slowly. Adding a few seconds of dilated time makes little difference if the age gradient does all the heavy lifting.

Another helpful concept, one outside the scope of this paper, is to think of inertial frames not as abstract spaces but as excited linear-momentum energy states of entities that originated in an earlier frame. Recognizing asymmetric energy excitation origins allows all clocks in the universe to be part of a single frame-origination tree.

An Age Gradient Example

Problem: A spaceship launches at 99.5% lightspeed — that is, at $0.995\,c$ or, equivalently, at $\beta = 0.995$ — by acquiring about nine times its rest mass in momentum energy. After launch, the ship's clocks run at about ⅒ its pre-launch speed, and its Lorentz compressed length is about ⅒ its pre-launch length. The launch also alters the ship's *interpretation* of the outside universe, which is unchanged energetically. To the ship, the length of the universe along the travel axis becomes ⅒ its pre-launch length.

(a) What age gradient does the ship see?

(b) When the ship arrives and stops one year later at its final destination 1 year, how much time has passed on the clocks at the destination?

(a) First, find the age gradient α_u of the universe that the ship observes:

1. Find: $\boldsymbol{\beta} = v/c = \mathbf{0.995}$, $\boldsymbol{\gamma} = 1/\sqrt{1-\beta^2} = 1/\sqrt{1-0.995^2} = \mathbf{10.0125}$, $\mathbf{1/\gamma} = \mathbf{0.099875}$

2. Apply the β version of the universal age gradient equation:

$$\boldsymbol{\alpha_u} = \frac{\beta}{c\sqrt{1-\beta^2}} = \frac{0.995}{(1\ ly/yr)\sqrt{1-0.995^2}} = \mathbf{9.9625}\ \boldsymbol{yr/ly}$$

(b) Find the destination clock time.

1. By its metrics, the ship travels $s = 1\ \mathrm{lightyear}$. The cumulative age traversed, α_Δ, is:

$$\boldsymbol{\alpha_\Delta} = \alpha_u s = (9.9625\ yr/ly)(1\ ly) = \mathbf{9.9625}\ \boldsymbol{yr}$$

2. All universe clocks dilate to $\gamma^{-1} = 0.099875$, so over one ship-year, they only add:

$$\boldsymbol{\gamma^{-1} t_\Delta} = (0.099875)(1\ yr) = \mathbf{0.099875}\ \boldsymbol{yr}$$

The total elapsed universe time when the ship arrives at its destination thus is:

$$\alpha_\Delta + \gamma^{-1} t_\Delta = 9.9625\ yr + 0.099875\ yr = \mathbf{10.062375}\ \boldsymbol{yr}$$

Discussion

The most critical question for age gradients is: *Why bother?*

After all, age gradients are nothing more than a more detailed way of labeling the lines showing Lorentz contraction in special relativity figures. They truly hide in plain sight.

However, there are several points worth considering:

(1) *Not* showing age gradients tends to confuse students and professionals. Any special relativity textbook or illustration that shows a Lorentz contracted object implicitly shows an object with an age gradient of $-\gamma\beta/c$ in the direction of motion. Not saying anything about this aspect of the Lorentz contraction makes it easy to assume the object shown *is* internally synchronous, which is never true.

(2) While gradients are relativistic and thus trivial to eliminate by switching to the object's point of view, this method fails when *two* age gradients are involved, such as in relativistic particle collisions. The resulting errors are subtle ones related to the states of the particles at the time of the collision and, for most scenarios, are exceedingly small. However, they are still tangible and testable.

(3) Requiring internally asynchronous objects to give standard physics turns out to be non-trivial. If $R = \sqrt{(c+v)/(c-v)} = e^w$, where w is rapidity, and $\theta = 0$ is the frame motion direction, then depart-return velocity pairs $\left(cR^{-\cos\theta},\ cR^{-\cos\theta+\pi}\right)$ replace the traditional idea of a single light velocity c. The invariant product of each pair is c^2.

(4) Noting that single point-of-view relativity with age gradients *requires* reciprocal light velocities is intriguing but likely not the end of the issue. There is a possibility that more complex patterns could have connections to more complex behaviors, such as quantum field theory.

(5) Apart from such more exotic considerations, age gradients are a better way to teach special relativity. Letting students know that the time it takes a spaceship to cross a galaxy converts into an age gradient helps them understand quickly and intuitively why slowing all of the clocks in the galaxy creates no time paradoxes. Age gradients complete the trio of Lorentz time dilation and Lorentz length contraction, and this completion provides a cleaner, more accessible way for students to understand a broad range of special relativity problems.

Finally, a note regarding the naming conventions:

While α (alpha) sometimes occurs in special relativity discussions to represent $1/\gamma$, this is a sufficiently minor use to permit the definition of a new meaning. The dominant use of α in physics represents the fine-structure constant, but most special relativity problems do not involve this constant. In cases requiring both the fine-structure constant and age gradient, the subscripted α_s standard age gradient distinguishes it from the constant.

Below are the four variants of the age gradient notations, along with the special relativity values used to construct them.

alpha-beta-gamma cheat sheet: $\alpha = -\frac{\gamma\beta}{c}$ $\beta = \frac{v}{c}$ $\gamma = \frac{1}{\sqrt{1-\beta}}$

$\boldsymbol{\alpha} = \boldsymbol{\alpha_s} = -\frac{\gamma\beta}{c}$ <u>Standard age gradient</u> observed in the forward direction of an object moving through rest.

$\boldsymbol{\alpha_u} = -\alpha = \frac{\gamma\beta}{c}$ <u>Universal age gradient</u> observed from a moving object in the larger frame it must traverse.

$\boldsymbol{\alpha_{\sim u}} \cong \alpha_u = \frac{\gamma}{c}$ <u>Approximate (pseudo) universal age gradient</u> for use with objects moving close to c.

$\boldsymbol{\alpha_m} = -\frac{\beta}{c}$ <u>Moving-frame age gradient</u> for finding the age gradient of objects with known rest lengths.

2022-09-21.00.00 EDT Wed *[Updated 2022-09-23] [Tweaked 2022-09-27]*
PDF: https://sarxiv.org/apa.2022-09-21.0000.pdf