Rapidity Defines Two Asymmetric Light Speeds in Special Relativity

Terry Bollinger
2022-09-12.03:35 EDT Mon

Email Excerpts

SUMMARY: The particle physics concept of *rapidity* — which provides a convenient way of representing extreme relativistic velocities as numbers that can grow indefinitely large and thus no longer "pack together" at the $\pm c$ limit — also defines an interesting symmetry of spacetime: the light-loop or R symmetry, in which the physics of an isolated inertial system remain unchanged when the forward and backward velocities of light diverge, subject only to the constraint that their product equals c^2. For a given rapidity $-\infty < w < +\infty$ along any selected xyz space axis, these velocities are $c_{\theta=0} = e^{-w}c$ and $c_{\theta=\pi} = e^{w}c$. The existence of such asymmetric light velocities not only preserves all known physics, but is required for special relativity to work. This Note also summarizes how a discussion with philosopher Ron Green, author of *Time to Tell* and *Nothing Matters: A Book About Nothing*, led to making an interesting connection between rapidity and asymmetric light speed pairs. Since this was a casual discussion, not a paper, flaws and insights are both recorded as they occurred.

Mon, Sep 12, 2022, 3:35 AM EDT

Hi Terry,

I have been thinking (which will be my downfall). It's about the speed of light. I was prompted by something I had come across countless times but reading it again (in Sabine's new book), my thought was triggered by our (your and my) present discussion about 'nothing'.

The speed of light is the same for all participants. [I use 'participants' instead of 'observers'. It's a long story, and I did mention in a previous email my problem with 'the observer', which I use for particular instances.] The caveat for the speed of light is that it refers to it being in a vacuum. As far as I can see, the "in a vacuum" is a way of saying that there is no impediment. It is assumed, then, that "a vacuum" has nothing in it. It's pure 'nothing', which is how you suppose. Now since I don't think that absolutely 'nothing' is possible, the speed of light can never be measured, and is entirely theoretical as a mathematical (physical?) 'fact' that works.

I am wondering whether Einstein thought there was such a thing as a vacuum that consists of absolutely nothing. Of course, it makes no difference as to whether a pure vacuum 'exists', but it is an interesting addendum to our discussion.

Best wishes,
Ron

Mon, Sep 12, 2022, 6:50 AM EDT

Hi Ron,

The later General Relativity Einstein definitely thought of space as "something" that could be curved. He even commented once (a bit notoriously) how taking his curved-space concept to its mathematical limit resulted in what amounted to a new, subtler version of an aether.

It's harder to say whether the Hume-inspired, pre-Minkowski 1905 Einstein believed in the vacuum as a kind of absolute nothingness. I don't think he did, mostly because his phrasing is more in terms of each observer having his own equal view of what the "space" of electromagnetic wave propagation is.

On the other hand, it's likely that in his Hume phase, Einstein was poking around at some quite profound possibilities for revising what space even is. This shows up best in his 1905 caution about whether a one-direction-only speed of light even has meaning. He wrote an equation in which the only thing one for sure is that the *average* I calculate for the forward-and-backward light speeds of a light pulse I send out to a mirror is c. He recognized that even the speed of light in the vacuum is so observer-centered that it has no meaning until the *same* observer sees the *same* light. Very Hume, that.

The YouTube Veritasium (Derek Muller) channel once nicely brought up this point about Einstein's caution on trying to define a one-way lightspeed, though all the stuff about ½ c and infinity as the two limits is non-relativistic and thus irreparably wrong:

> *Why No One Has Measured the Speed Of Light*
> https://youtu.be/pTn6Ewhb27k?t=6m43s

As I discuss below, the real limits are zero and infinity after you make the idea relativistic by adding in Lorentz contraction and time dilation.

Incidentally, you, too, are falling into the subtle trap of assuming the physics of physics can exist independently of the observer when you talk about c, the speed of light, being the "same" for all observers. It's not even definable.

Here's what really happens: For any one observer, a new constant, $C = c^2$ (capital C equals the speed-of-light c squared) is the *only* invariant. As long as the product of the out-and-back light speeds equals C, that is, equals c^2, it works. Observers in other frames can correctly say your effective light speed ranges from approaching-infinity in one direction to approaching-zero in the other direction, provided only that the product of the two is C.

When this pair of lightspeeds that you see in some other inertial frame is extremely off-balance, you get Lorentz contraction and time dilation.

If you are curious about the remarkably and literally self-centered, but also more physically accurate, $C = c^2$ definition of lightspeed, it's the fourth-from-last equation on the last page of this Note:

Why the Age Gradients in Special Relativity Are Easily Overlooked
https://sarxiv.org/apa.2022-09-03.1953.pdf

Incidentally, over the course of writing this I've watched my smartphone-based access to my own website, both by search and by direct link name, quite literally disappear. It looks transient, but just to warn you, it's possible you'll have trouble accessing the above link. I'll look into it later this morning.

Cheers,
Terry

Mon, Sep 12, 2022, 7:29 AM EDT

Hi Terry,

I HAD TO reply immediately because of your justified slap on my wrist about my "the speed of light is the same for all participants". My bad. Thank you for pointing it out. I copied it from Sabine's new book without thinking about it too much. The sentence would have been more judiciously put: "the speed of light is the same for each participant, depending on his location and velocity". Is that better? Not yet good enough, I fear.

Best wishes,
Ron

Mon, Sep 12, 2022, 9:50 AM EDT

Hi Ron,

It's a bit worse than just saying lightspeed is always personal.

I'm saying — and to be honest, this is just repeating what Einstein said — that the idea that light has one velocity is bogus. Light speed can be anything it wants to be, as long as its product times lightspeed in the *opposite* direction equals c^2, which I've take to calling the loop constant $C = c^2$.

Here's what that means: Whenever you look in a mirror, the light from your face *could* travel at nearly infinite speed *towards* the mirror, then take a billion years to get back to you. As long as the product of those two velocities is C, the physics of the situation remains invariant. The *only* value guaranteed in all of this is the loop constant $C = c^2$.

You cannot detect these different possible lightspeeds, ever. However, anyone moving differently from you will *always* see reciprocal lightspeeds for your frame, and vice-versa.

The cases in which the return light takes a billion years are what the other frame calls extreme time dilation.

This behavior is intrinsic to the more precise special relativity equations that particle physicists use every day. Nonetheless, it probably counts as a new finding since quite a few papers talk about the to-from speed problem but gets the numbers wrong due to treating space distances as absolutes. That's never true in our universe, and the error results in a bogus ½ c minimum speed. Light can instead go *indefinitely* slow.

…

Cheers,
Terry

Wed, Sep 14, 2022, 1:52 AM EDT

Hi Terry,

Thank you so much. I get it. I find the *Why No One Has Measured the Speed Of Light* (https://youtu.be/pTn6Ewhb27k?t=6m43s) fascinating. So, we can't measure light in only one direction. Yes, I see the connection/problem with the observer.

But something puzzles me. Isn't it a fundamental element of SR that the speed of light is a constant, the speed of light is finite? If so, what do you mean by "Light speed can be anything it wants to be" and "Whenever you look in a mirror, the light from your face *could* travel at nearly infinite speed *towards* the mirror, then take a billion years to get back to you." What am I missing?

BTW, I have read again your *Einstein, Hume, and Mathematical Creationism* at sarxiv.org/apa and I (again/still) think it's wonderful.

In case you are wondering why I am going on about all of this, which is, after all, physics... It's because there is a profound link with the thoughts that my proposed book are based upon. The observer, the observed, knowledge and reality, time... I'm struggling to hack my way through the undergrowth of mangled ideas and wonder if I can separate them in order to get to a coherent linear exposition. You physicists have it so easy [Ron ducks].

Best wishes,

Ron

Wed, Sep 14, 2022, 4:24 AM EDT

Hi Ron,

This is short, I′m still asleep. :)

$100\ m$ spaceship launches at 60% of lightspeed, $0.6\ c$. Captain Hume at the back of the ship launches a light pulse toward a mirror at the front of his ship at launch. His ship is running away from the pulse, so it takes longer to get there. The "effective" forward speed of the pulse is $1.0\ -\ 0.6$ or $0.4\ c$.

It bounces and returns, but now the ship is rushing toward the pulse to give an "effective" lightspeed of $1.0\ +\ 0.6$ or $1.6\ c$.

Einstein noticed this in his 1905 paper.

Must sleep now ..

Cheers,
Terry

Wed, Sep 14, 2022, 8:45 AM EDT

[continued]

The reason these "effective" $c + v$ and $c - v$ lightspeeds are more than illusions is that in SR no one can ever say they are the one and only "true" rest frame. That assertion has no meaning in SR. You may be the one moving at 60% light speed, not the ship, in which case *you* become the one whose lightspeed opposite the ship′s direction of motion becomes $c - v$ by the ship′s accounting and $c + v$ in the same direction as the ship′s motion.

There are two ways to measure these varying speeds. The rest frame sees the moving frame approach $0\ c$ speed its direction of motion, and $2\ c$ in the opposite direction, with both adding up always to $2\ c$.

However, if that same pair of speeds is expressed using only the clocks and rulers of the moving system, the limits become 0 and infinity, with the *product* of the two, rather than the sum, always being c^2, as opposed to $2c$.

These are better coordinates in the sense the rest frame can, if they wish, create and choose *any* moving frame by which to reinterpret their direction-dependent lightspeeds. Importantly, *all* such choices are guaranteed by SR to give the *same* physics.

More practically — more experimentally — it makes the following assertion true: If SR is correct, assigning light to an arbitrarily high speed in one direction and an arbitrarily low speed in the opposite direction always gives the same physics as assuming one lightspeed, c, in both directions. The only constraint is that the product of the forward and backward speeds chosen must equal c squared. There's a bit more to it, of course, since there's also a smooth cosine function for defining lightspeed pairs for angles other perfectly forward.

So why don't we notice any of this? Mostly because our huge rest frames — e.g., earth — make the single-c approximation extraordinarily effective for most situations. Like most things in special relativity, you need pretty extreme physics for the explicit use of reciprocal lightspeeds to become relevant. Importantly, even in those extreme cases, you don't *have* to use reciprocal lightspeed pairs. I would argue that ignoring them is risky, though, since these pairs create non-simultaneous particle states that no change in the viewer frame can eliminate.

This kind of complete indifference to an experimentally meaningful parameter — I call it R, the ratio of forward distance covered by light for the moving object and the same object at rest — is a good example of a physics symmetry.

The R symmetry — the impossibility of proving light is nearly infinite in one direction and nearly zero in the other — is necessarily implied by the Poincaré symmetries. However, I've yet to encounter a name for it. That may be because papers that address Einstein's original 1905 observation keep getting its structure wrong by assuming space to be invariant in length. That gives a bogus symmetry in which the minimum lightspeed appears to be ½ c instead of $0\,c$.

Given the applicability of the R symmetry when describing *any* physical event is why c^2 is the only theoretically meaningful constant.

Finally, it's interesting how incredibly observer-centric the R symmetry is. The only way to measure c^2 is by creating a light loop that goes out, probes an object, and returns. Even with such a loop, there's no way to tell how distant the object is in time. It could take a billion years for light to cross your living room, yet unless some larger context gets tired of waiting on you, you'll never know it.

Cheers,
Terry

Thu, Sep 15, 2022, 8:16 AM EDT

Hi Terry,

… Could you please elucidate for me what the "effective speed" of light is. Effective as opposed to...?

Best wishes,
Ron

Thu, Sep 15, 2022, 10:01 AM EDT

Ron,

I wanted to thank you for poking me on that issue since it provoked me into expressing it as a mathematical symmetry. I realize they won't be helpful, but I'll send the equations later. They're another way of expressing the ancient Poincare symmetries, but it's an interesting way:

Short version: There are three parameters $0 < R_x, R_y, R_z < +\infty$ that describe reciprocal lightspeed components... *Ah!* The base-4 logarithm of R, which I call rho (ρ), works better here!... Restarting: There are three parameters:

$$-\infty < \rho_x, \rho_y, \rho_z < +\infty$$

... that define all possible "lightspeed-pair spheres" that give *identical* physics. Cool... rho is *way* cleaner here... thanks...

Anyway, "effective" means that from the rest-frame view it's *always* safe (but *never* necessary) to assume one lightspeed c in all directions. For rest, the spaceship uses that same c, but also moves in a race with the light, slowing its arrival. That means that "effectively," the speed of light crossing the length of the spaceship forward is slower since the ship crew sees only the back-to-front length of their ship, not the added travel distance. Similarly, light moving backward hits the back of the ship faster, giving faster effective lightspeed.

The catch is that if you instead choose to interpret this "effective" delay as an entirely real "in-frame" speed, there is no test you can do to prove otherwise. That has to be true, or else the spaceship would not see the same physics that we see.

The bottom line is that our choice to say that speed of light is the same in all directions is not supported by physics. The physics says that all you need are pairs of light that have the same product. It is always valid to say that the light from your face goes to the mirror at a speed close to infinity, then comes back at an incredibly slow snail's pace.

As long as the product of those two is c^2, it is a perfectly valid description of the physics of you looking in the mirror.

As long as you follow the full math for the entire 3-globe of lightspeed pairs, choosing asymmetric values for light speed is every bit as valid as saying that c is identical in both cases. Saying c is identical, however, is an extremely narrow special case of the actual physics going on. There is no *one* velocity of the speed of light, and that is a fundamental requirement of special relativity, not an accident.

Cheers,
Terry

Sun, Sep 18, 2022, 11:49 AM EDT

Ron,

I said:
>… "There are three parameters $0 < R_x, R_y, R_z < +\infty$ that describe reciprocal lightspeed components… *Ah!* The base-4 logarithm of R, which I call rho (ρ), works better here! … Restarting: There are three parameters $-\infty < \rho_x, \rho_y, \rho_z < +\infty$ that define all possible "lightspeed-pair spheres" that give *identical* physics. Cool… rho is *way* cleaner here… thanks…"

This keeps getting more interesting! Thanks again, as it's helped me connect my geometric values to particle physics.

My $\rho = 1.4427w$, where w is a physics parameter called *rapidity*. When particle physicists use rapidity, they label it, rather confusingly, as y, even though they also use y for xyz coordinates. Rapidity goes to $\pm\infty$ and thus is easier to use than velocity for extremely relativistic velocities where everything starts looking like a long row of 9s.

My R and ρ parameters are purely geometric. Both describe a rectangle that stretches out as speed increases but whose volume stays invariant no matter how thin the rectangle becomes. I designed the rho version to behave like a velocity, that is, $v = 0$ and $\rho = 0$ both mean the object is stationary, negative v and ρ both mean it's moving backward, and positive v and ρ both mean it's moving forwards.

The difference, again, is that velocity can never get smaller than $-c$ or larger than $+c$, while ρ can go to plus or minus infinity like an ordinary coordinate. The limits on velocities mean extreme velocities get "squished" (pardon the technical jargon) at highly relativistic velocities, making it hard to eyeball the difference between $0.999999\,c$ and $0.9999999\,c$. More importantly, such numbers are harder to use on computers due to wasting so many digits on repeating 9s that don't say much.

Here's the conversion table I got after recognizing that rho is just a multiple of rapidity. It's in Google equation format here, which makes it easy to calculate values. Beta is the velocity in units of speed-of-light, e.g., $0.6\,c$:

```
        R: ((1+(beta))/(1-(beta)))^(1/2) =
   w=ln(R): ln(((1+(beta))/(1-(beta)))^(1/2)) =
       or: (1/2)*(ln((1+(beta))/(1-(beta)))) =
 rho=f(w): ((ln(e^2))/(ln(2^2))*(ln(((1+(beta))/(1-(beta)))^(1/2))) =
       or: (2/ln(4))*(ln(((1+(beta))/(1-(beta)))^(1/2))) =
       or: (1/ln(4))*(ln(((1+(beta))/(1-(beta))))) =
```

Or, in LaTeX format:

$$R = \sqrt{\frac{c+v}{c-v}} = \sqrt{\frac{1+\beta}{1-\beta}} = \left(\frac{1+\beta}{1-\beta}\right)^{½}$$

$$w = \ln R = \ln \sqrt{\frac{1+\beta}{1-\beta}} = \ln\left(\left(\frac{1+\beta}{1-\beta}\right)^{½}\right) = ½ \ln\left(\frac{1+\beta}{1-\beta}\right)$$

$$\rho = \left(\frac{\ln(e^2)}{\ln(2^2)}\right) w = \left(\frac{\ln(e^2)}{\ln(2^2)}\right) \ln\left(\left(\frac{1+\beta}{1-\beta}\right)^{½}\right) = \left(\frac{2}{\ln 4}\right) \ln\left(\left(\frac{1+\beta}{1-\beta}\right)^{½}\right) = \left(\frac{1}{\ln 4}\right) \ln\left(\frac{1+\beta}{1-\beta}\right)$$

The factor $\ln e^2 / \ln 2^2$ converts e^2 area units to 2^2 units, giving $R = 2^\rho$ versus $R = e^w$.

Below are the R, ρ, and w number for my earlier two examples of similar-looking velocities:

$$v = 0.999999\, c \quad R = 1414.2132088 \quad w = 7.25432861925 \quad \rho = 10.465783924$$
$$v = 0.9999999\, c \quad R = 4472.13584437 \quad w = 8.40562139102 \quad \rho = 12.1267482964$$

Apologies for all the math, but this was a mostly real-time chain of thought, and I needed to get it down. You just happened to be the poor person getting hit with all of it... :)

Cheers,
Terry

Sun, Sep 18, 2022, 12:06 PM EDT

... and, getting back to speeds of light issue, the forward and backward light speed in any chosen direction, this time in terms of particle physics *rapidity*, w, are [1]:

$$c_{forward} = c_{\theta=0} = R^{-1}c = e^{-w}c$$
$$c_{backward} = c_{\theta=\pi} = Rc = e^{w}c$$

The R can be any number from minus infinity to plus infinity, yet you end up with the same physics. As rapidity w increases, forward lightspeed gets slower (down to zero velocity) while backward lightspeed gets faster (up to infinite velocity). However, within the frame, all settings of w give the same physics. How beautifully simple!

Cheers,
Terry

[1] These are the equations *after* two corrections at 12:23 PM EDT and 2:08 PM EDT

Sun, Sep 18, 2022, 5:44 PM EDT

Hi Ron,

And after all of that... I think I can now get back to your *real* question:

>… Isn't it a fundamental element of SR that the speed of light is a constant, the speed of light is finite? If so, what do you mean by "Light speed can be anything it wants to be" and "Whenever you look in a mirror, the light from your face could travel at nearly infinite speed towards the mirror, then take a billion years to get back to you." What am I missing?

The short answer is that if *everyone's* clocks are delayed in one direction by a billion years, it makes no difference. Everyone's clocks read the same and tick the same, so no one in that inertial frame can detect the incredibly prolonged delay imposed by the slow part of the light cycle. The fact that one *cannot* say the actual light speed in either direction is precisely why this is a deep symmetry of spacetime. As long as only one inertial frame exists, *all* possible R values are equally possible.

However, this is where it gets interesting: One *can* break the symmetry and make it experimentally detectable.

To break the light-loop or R symmetry and make it experimentally detectable that it takes a billion years to travel either to or from the mirror, the trick is to add *another* inertial frame and somehow "move" (!) the observer to that frame. When this breaking of who is doing the observing occurs using an observer in a relativistic inertial frame, we call it time dilation. But *within* the original frame, no paradox occurs since there is no way to detect the asymmetric light speed using *only* one frame.

Thus if a very fast spaceship passes through our solar system, we observe the "backward tick " of any clock in the ship — the part of the motion of a clock or clock signal that moved opposite of the spaceship's motion — as occurring incredibly quickly by our standards. In contrast, its "forward tick" is unbelievably slow. Add the two half-ticks together, and you get the Lorentz time dilation.

All this is in the Poincare symmetries, so this is nothing new. For example, if you express the Lorentz factor in terms of R, which is proportional to the duration of the longer half-tick and inversely proportional to the duration of the shorter half-tick, the Lorentz factor is the average of the two: $\gamma = (R + R^{-1})/2$.

It's hard to make this concept intuitive, however. The idea that "half" of a dilated clock tick occurs incredibly *faster* than our own clocks just doesn't sound right, so I suspect even physicists don't often think of the situation that way. At very high R values, the delay caused by the long half-tick is so enormously larger than the speed-up caused by the short half-tick that the short tick can be ignored entirely. That is the basis for quantity particle physicists call pseudorapidity, which is "almost" rapidity but easier to calculate due to it ignoring the backward clock-tick. That's not the usual definition of pseudorapidity, of course, but it's useful to keep in mind.

Finally, I'm skipping over a critical point: When the R symmetry is broken, it requires *energy*. The more severely the two-way light speed symmetry is broken, the higher the energy cost — and that part of the cost is never relative. If an observer in another frame sees your image traveling to a mirror almost instantly but then takes a billion years complete the loop, it means that an incredible amount of energy was invested in either you or the other system to accelerate it. There's nothing relative about this energy investment: It either happened, or it did not. Also, for actual experiments, it's almost always best to make the rest frame the one in which such energy was *not* invested.

Energy investment is also where all of this makes contact with General Relativity since the energy invested in a local instance of an inertial frame is a real excitation of the matter in that frame and thus adds to the gravity of that matter. No surprise there, but consider this: If the R symmetry of light-speed loops is a deep part of spacetime, why is gravitational mass breaking it and making one frame different from another?

Enough, I've gone on far too long in all of this. I just purchased and downloaded a copy of Sabine Hossenfelder's *Existential Physics*. The early figures gave a nice intro to some SR issues, so if you have specific questions, please let me know, and I'll see if I have any helpful insights.

Since this has turned into a nice example of on-the-fly recognition perusal of some interesting special relativity math issues, I'll be posting a cleaner version that includes a conversation of at least some of the formulas in LaTeX format.

Cheers,
Terry

Terry Bollinger
2022-09-12.03.35 EDT Mon
PDF: https://sarxiv.org/apa.2022-09-12.0335.pdf