Why the Age Gradients in Special Relativity Are Easily Overlooked

Terry Bollinger

2022-09-03.19:53 EDT Sat [updated 2022-09-06] [corrected 2022-09-27][alpha names tweaked 2023-02-03]
https://profmattstrassler.com/2022/09/01/relatively-confused-is-it-true-that-nothing-can-exceed-light-speed/#comment-448556

Expansions (see p. 3-4) and fixes to comment on *Of Particular Significance* (Matt Strassler) post: *Relatively Confused: Is It True That Nothing Can Exceed Light Speed?*
https://profmattstrassler.com/2022/09/01/relatively-confused-is-it-true-that-nothing-can-exceed-light-speed/

Hi Matt Strassler: I don't think I can answer your most recent questions without explaining more precisely what I mean by "age gradients" since, at this point, I suspect there is no existing terminology for this idea after all. As usual, please feel free to truncate or delete this long answer as you see fit. Just so you know, I'm also posting a copy on my website, regardless of whether you keep or delete your copy.

Age gradients, $\alpha = -\beta\gamma/c = -\beta/\left(c\sqrt{1-\beta^2}\right)$ in rest-frame meters or $\alpha' = -\beta/c = -v/c^2$ when applied to in-frame meters, apply at all special relativity scales. So let's forget particles for the moment and talk about human-scale clocks to get a feel for why I think age gradients could result in measurable changes in the state of colliding particles.

Imagine two clocks, A on the left and B on the right, $100\,m$ apart ($\ell = 100\,m$) on the z-axis and networked by lasers [1]. If the clocks accelerate independently, what steps ensure the clocks keep the same space-and-time relationship after boosting into the $v = 0.6\,c$ frame where A becomes the trailing clock and B becomes the leading clock?

(1) Move the clocks $20\,m$ closer to prepare for $\ell/\gamma = 100\,m/1.25 = 80\,m$ Lorentz compression.

(2) Reset the clocks from showing identical times (synchronous) to showing times where A is $200\,ns$ ahead of B (asynchronous or *time-tilted*). The new mix of space and time in the moving frame m is the source of this change. The end-to-end difference for $v = 0.6\,c$ is $\Delta t = \alpha'\ell = (-\beta/c)\ell = (-0.6/c)(100\,m) = -200.14\,ns$.

(3) Wait if you want to. Boosting a complex system into a new inertial frame is a multi-step process with many solutions. Compression, time-tilting, and acceleration are the three conversions needed, and these can occur in any of eight possible orders. Since compression and time-tilting are deltas, there are also infinitely many ways to change the clock positions and clock times to achieve the needed deltas. In short, when analyzed carefully, acceleration is a messy process that entails far more than just "pushing" an object.

(4) Complete the trio of transformations by implementing the final step of, in this case, "instantly" accelerating both clocks to $0.6\,c$. The resulting system stays compressed and time-tilted in the rest frame but is now internally synchronous with $100\,m$ distance in its new $0.6\,c$ frame. While it sounds intuitively impossible that a system that is provably asynchronous in the rest frame can nonetheless look synchronous

> in the moving frame, the finite propagation speed of light ensures that the moving clocks cannot devise any experiment that informs them which direction has "already occurred" and which direction has "yet to occur." By the time light signals arrive at a single point in the moving frame, it is guaranteed not to matter.

The differently tilted time slopes of colliding systems make it impossible for both to be synchronous at the point of collision since only one can be made synchronous by changing the point of view. Both objects are asynchronous in the case of a rest-frame collision of two relativistic objects with opposite and equal rest-frame velocities.

Apart from collision physics, age gradients also trivially explain many seeming space travel paradoxes. The *traveler's age gradient* $\alpha_+ = -\alpha$ describes the age gradient that a newly launched traveler encounters as she travels forward within that larger pre-existing frame. For example, a fast space traveler encounters a Lorentz-compressed universe with a positive age gradient $\beta\gamma/c$ in the direction of travel. This age gradient "encodes" what others in the universe interpret as time passing while she travels, with both arriving at the same result when she arrives. If a traveler's velocity is very close to c, that is, if $\beta \cong 1$, the *approximate (pseudo) traveler's age gradient* $\alpha_\sim = \gamma/c \cong \alpha_+$ is easier to use.

Twin paradoxes emerge when symmetric time dilations are confused with asymmetric age gradients. Time dilations always add time *symmetrically* to both frames, with the size of this contribution dropping rapidly as the velocity difference approaches c. In contrast, the boosting process creates age gradients asymmetrically. The original frame experiences no such boost and thus sees no age gradient. Its view of space remains synchronous. However, the age gradient is real to the moving object, and once it traverses it, there is no going back. The time has elapsed in the earlier frame and is no longer recoverable.

The 1991 Oh-My-God (OMG) particle is one of the most extreme known cases of such an asymmetric age gradient. Assuming it was a proton, its Lorentz factor was 3.2×10^{11} and its approximate universal age gradient was $\alpha_\sim = \gamma/c = 1067\ s/m$. By flying directly into such a steep gradient at a velocity very close to c, it began traversing $10{,}140$ years of universe time for each second of its own time. For an idea of how extreme such an age gradient is, if the OMG particle began its flight 13.7 billion years ago with the universe's birth, it was only 15.6 days old when it hit earth's atmosphere.

That *looks* like time dilation, but it is not. From the perspective of the proton, all clocks in the outside universe advanced by a mere 3.125 picoseconds per second. That's a pretty negligible contribution to total elapsed time in comparison to the $10{,}140$ years added each second by traversing $299{,}792{,}458$ meters of a $1067\ s/m$ age gradient.

Another factor beyond this discussion is the importance of "launch" or *excitation* hierarchies," that is, the actual history of how a system came to be in a particular frame Special relativity problems often skip over how a system came to be in its current inertial frame. Age gradients emphasize that this is a bad idea since only the system that was *not* excited into a higher-energy and higher-momentum state can safely interpret higher frame states as *non-relative* phenomena in their systems.

Age gradients point out the need to track the *origins* of boosted objects since each lower frame in the resulting tree of boosts can always view higher-frame dilations as real.

While all this discussion has been about large objects, age gradients also provide a different and more straightforward way of interpreting the complex-plane phases of Schrödinger momentum wave functions: They are age gradients. Rest-mass quantum frequencies acquire age gradients in the direction of motion, twisting their amplitudes into a helix. Wavefunctions, thus, are necessarily an aspect of special relativity, but this only becomes apparent after recognizing the impact of age gradients on wave function phases.

I'm sorry if I'm introducing new physics in any or all of this. I was sincere when I said I'm just trying to figure out the names of some of this stuff. When I find something interesting — Glashow's cube is an example — I assume it's well-known, and I don't yet know the keywords to look it up. But if there are no existing keywords, then maybe it is new.

But why would such an almost absurdly simple concept get overlooked for over a century?

Increasingly, I suspect it's because the Poincaré symmetries work so well that the instant we say "object," our brains switch over to moving-frame space and time. Why not? They are, after all, absolutely valid coordinates. Thus the fact that we changed frames doesn't even occur to us.

The danger of that seductive little mental flip is that it undermines how fiercely SR applies in every situation. SR applies even when it converts ordinary objects into weirdly time-sloped entities whose internal light speeds vary by direction, with an effective forward light speed of cR^{-1} and backward lightspeed of cR. The idea that lightspeed varies by direction is not new, but the papers I've seen on it assume minimums and maximums of ½ c and infinity. The ½ c minimum is just wrong since space is always relative.

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[1] The two-clock system details are: Both clocks use Doppler to stay at rest relative to the other. Both use $667\,ns$ $(200\,m)$ send-return signal loops to ensure $100\,m$ separations. Both send their current times in each send-return loop, and both synchronize their clocks based on the other clock being $333.6\,ns$ distant.

ADDENDUM 2022-09-06.12:30 EDT Tue

This update defines the several variants of the age gradient, the light-paths ratio R and its relationship to rapidity, and the curious relationship between asynchronous objects and reciprocal light speed pairs.

The alpha-beta-gamma "cheat sheet" for remembering the standard age gradient is:

$$\alpha = -\frac{\beta\gamma}{c} \qquad \beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

In the equations below, *forward* is the velocity direction z^+, *backward* is z^-, *transverse* is any vector in the moving-frame xy plane, *positive* is any light vector outbound at an angle θ, and *negative* is the opposite, inbound, or return light vector for that θ, e.g., after a mirror reflection.

Standard age gradient α is the age gradient seen from rest inside moving objects:

$$\boldsymbol{\alpha} = -\frac{\beta\gamma}{c} = -\frac{\beta}{c\sqrt{1-\beta^2}} = \frac{v}{c\sqrt{1-v^2/c^2}} \quad \textit{where} \quad \boldsymbol{\beta} = \frac{v}{c} \quad \textit{and} \quad \boldsymbol{\gamma} = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-v^2/c^2}} \tag{1}$$

Traveler's age gradient α_+ is the age gradient a moving object observes around itself:

$$\boldsymbol{\alpha_+} = -\alpha = \frac{\beta\gamma}{c} = \frac{\beta}{c\sqrt{1-\beta^2}} = \frac{v}{c\sqrt{1-v^2/c^2}} \tag{2}$$

Approximate (pseudo) traveler's age gradient $\alpha_\sim$ is useful when $\beta \cong 1$ (near-c velocities):

$$\boldsymbol{\alpha_\sim} = \frac{\gamma}{c} \cong \alpha_+ \tag{3}$$

In-frame age gradient α' is a shortcut for an object with a known in-frame (rest) length ℓ:

$$\boldsymbol{\alpha'} = -\frac{v}{c^2} \tag{4}$$

Forward light-path length ratio R is moving-frame path length over rest-frame length:

$$R = \sqrt{\frac{c+v}{c-v}} = \left(\frac{c+v}{c-v}\right)^{½} \tag{5}$$

Positive (outbound) light velocity c_θ for any cone angle θ measured from forward z^+ is:

$$c_\theta = cR^{-\cos\theta} = c\left(\frac{c+v}{c-v}\right)^{-½\cos\theta} \tag{6}$$

Negative (inbound or reciprocal) light velocity $c_{\theta+\pi}$ for the same cone angle θ is:

$$c_{\theta+\pi} = cR^{\cos\theta} = c\left(\frac{c+v}{c-v}\right)^{½\cos\theta} \tag{7}$$

Loop invariant C for any positive-negative (outbound-inbound) light propagation pair is:

$$C = c_\theta c_{\theta+\pi} = \left(c\left(\frac{c+v}{c-v}\right)^{-½\cos\theta}\right)\left(c\left(\frac{c+v}{c-v}\right)^{½\cos\theta}\right) = c^2\left(\frac{c+v}{c-v}\right)^{-½\cos\theta+½\cos\theta} = c^2\left(\frac{c+v}{c-v}\right)^{0} = c^2 \tag{8}$$

Forward light velocity for z^+ is:

$$c_{z+} = c_{\theta=0} = c\sqrt{\frac{c-v}{c+v}} \tag{9}$$

Transverse light velocity for xy is:

$$c_{xy} = c_{\theta=\pi/2} = c \tag{10}$$

Backward light velocity for z^- is:

$$c_{z-} = c_{\theta=\pi} = c\sqrt{\frac{c+v}{c-v}} \tag{11}$$

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2022-09-03.19:53 EDT Sat [updated 2022-09-06] [corrected 2022-09-27][alpha names tweaked 2023-02-03]
PDF: https://sarxiv.org/apa.2022-09-03.1953.pdf