The Role of Meter-Seconds in High-Precision Special Relativity

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Meter-seconds are the units of one of three relativistically invariant values used to specify the precise location of an event in spacetime. It is a surface area measure that directly captures the ambiguity of how the *less* fundamental, observer-dependent metrics of length and duration emerge in a given situation. The meter-second thus qualifies as one of the most fundamental units of special and general relativity, despite its lack of broad recognition.

I've never encountered a standard name for this measure, so I'll name it by describing it. It is the length-duration, $\Delta x \cdot \Delta t$, or Lorentz area L, of an event in spacetime. If x is the axis of motion of the object relative to the observer's frame, Δx is the length of the region in which an event occurs, e.g., the back-to-front length of a moving clock. Δt is the duration of the activity that defines the event, e.g., one tick of the moving clock.

Calling it the Lorentz area $L = \Delta x \cdot \Delta t$ makes sense because the length of the event in the direction of travel is *divided* by the Lorentz factor, while the duration of the event is *multiplied* by the Lorentz factor. The two Lorentz factors cancel in the product, giving a meter-seconds area that stays invariant for all velocities. A notable feature of the Lorentz area is that even though its magnitude is constant, its shape is not. At rest, the base area unit of L is a square with an area of $1\, m \cdot s$. At high velocities, this square retains an area of $1\, m \cdot s$ but morphs into a rhombus. At the limit of an event occurring at speeds very close to c, the event rhombus approximates a ray on a light cone.

The other two relativistically invariant event location measures are the width Δy and the height Δz of the event, e.g., the width and height of the moving clock. These are perpendicular to x in xyz space and form a second area $P = \Delta y \cdot \Delta z$. In 4-space, the area P is also perpendicular to L, creating a 4-box. Like L, the P area is invariant in magnitude, but P does not change shape.

You've never heard about Lorentz areas or the importance of meter-second units because special and general relativity events are usually modeled approximately as points in a 4-space. This approximation, akin to the common practice of modeling a rigid mass as a single point, simplifies calculations by skipping over details that, in low-velocity cases, make little difference. Unfortunately, models of extreme velocity events typically try to use the 4-point approximation. That is a problem because mapping a 3-space event box $(\Delta x, \Delta y, L)$ into (x, y, z, t) 4-space creates stretched-out event rhombohedrons that more closely resemble rays in light cones. The point approximation in such cases usually refers to one rest-frame space-slice of this ray at a single moment in rest-frame time, which also gives the Lorentz length of the event at that time. Many of the time paradoxes of relativity arise from confusing these less complete but more point-like, single-time-only space slices of the event ray with the full ray.

Finally, accurate modeling of high-velocity event rays requires defining a shared xyz space by creating a large rest frame containing many synchronized objects, such as the machinery of a particle accelerator or the earth's atmosphere for cosmic-ray muons. Full specification of the event box within an embedding frame requires not one but three LP pairs, one for each xyz velocity component relative to the space-defining embedding frame. Thus, in sharp contrast to approximating an event as a single point in x, y, z, t 4-space, the full and unambiguous relativistic model of an event taking place within a well-defined embedding frame requires bounding areas in the six planes $\{xt, yt, zt, yz, zx, xy\}$. These areas are grouped into three LP pairs $\{\,(xt, yz), (yt, zx), (zt, xy)\,\}$. Notably, and not coincidentally, these same six planes define the relativistic components [1] of the electromagnetic field $E_x = F_{xt}$, $E_y = F_{yt}$, $E_z = F_{zt}$, $B_x = -F_{yz}$, $B_y = -F_{zx}$, and $B_z = -F_{xy}$.

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[1] Feynman Lectures, Vol. II, Table 26-1, The components of $F_{\mu\nu}$
https://www.feynmanlectures.caltech.edu/II_26.html#Ch26-S3-p8

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