Traditional Logic Versus Quantum Indistinguishability

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Email excerpt

https://sarxiv.org/apa.2022-07-02.1510.pdf

Mitch,

Fascinating discussion, thanks.

Have you ever heard of Schrödinger logic? In researching what you said, I just encountered it myself for the first time:
https://en.wikipedia.org/wiki/Schr%C3%B6dinger_logic

Oddly, most quantum folks are highly familiar with the principle of indistinguishability that underlies Schrödinger logic. Nonetheless, this principle's philosophical implications have largely escaped the attention of philosophers and logicians over the years.

The principle of indistinguishability in quantum systems is this: If you do not have distinct, experimentally meaningful state numbers for two particles of the same type, *nothing in the physical world can say which particle is which*. For example, any attempt to assign labels such as a and b to two electrons in the same atom is meaningless. All you can say is you have two electrons worth of quantum numbers in that atom.

Based on how indistinguishability plays out mathematically, you get two types of indistinguishable particles: bosons and fermions. Why two? Because — and I'm not making this up — positive-energy numbers have two square roots. You get a pair of bosons if the particles' square roots have the same sign ("symmetric"), and you get a pair of fermions if the particle's square roots have opposite signs ("antisymmetric").

Bosons like to be in the same state, and these are the ones that mess most wildly with the concept of unique identity.

For example, a metal-atom Bose condensate begins with a thin gas of metal atoms that, under the right conditions, suddenly pops into a new state that looks like a mysteriously floating perfect metal sphere. Once the metal atoms form such a condensate, there is *no test* by which you can distinguish one atom from another without breaking the condensate apart.

A Bose condensate is not a crystal. Every atom in the crystal has precisely the same wave function and thus location, filling in the *entire* human-scale sphere. Thus they profoundly share their identity. It's not even quite correct to say (as I just did) that the atoms in the condensate all have the "same" quantum wave function. There is just *one* wave function, which contains quantum numbers equivalent to lots and lots of atoms.

It's the other group of particles, the fermions, that in sufficient numbers enable us to start talking about concepts like "identity." But even there, it's a lot messier than folks realize.

Take, for example, the two electrons in a cold helium atom. They are fermions, so they require distinct quantum numbers. The two electrons in helium use a quirk called spin to distinguish themselves from each other. By doing so, they can occupy the same space around the helium nucleus, much as with the larger-scale Bose condensate of atoms.

After using up the spin distinction, the only way one pair of helium atom electrons can distinguish itself from a similar pair in some other helium atom is by the entire atom occupying a different location in xyz *space*. The ensuring concepts of volume and relative positions of atoms are the first steps in creating what we so casually call "unique" identities for large-scale objects.

However, even for a single helium atom, it's incorrect to label the two electrons as a and b. All you have is a fuzzy wave function containing two electrons' worth of quantum numbers. Only if you *push* that single wave function and ask it to distinguish between the two electrons can you begin distinguishing the two electrons — and by then, they are no longer in the original rest state. (A qualifier: I'm skipping over a further complication, which is the role of helium nucleus spin. The principles involved stay the same.)

The only way to label the atoms as a and b is to use a massive classical instrument to provide a well-defined spin *direction* in xyz space, specifically by applying a magnetic field. Once this field is created and applied to the atom, it finally becomes possible to assign physically meaningful labels to the two electrons of the helium atom: "up" in the magnetic field becomes electron a, and "down" becomes electron b.

Note that the definition of "up" must be *added* by some external classical instrument or other particles creating such a field. It is *not* by any inherent property of its electron wave function. Most textbooks skip over this issue, and that's unfortunate. The "up" and "down" directions of an electron orbital have meaning only in the presence of such a field. For all other cases, they are no more distinguishable than the atoms in a Bose condensate.

This inability to distinguish objects at the quantum scale wreaks havoc with traditional ideas of identity and distinguishability of objects. Yet oddly, until just now encountering this idea of "Schrödinger logic," I can't recall any discussion by quantum philosophers on how indistinguishability seriously undermines traditional concepts of unique identity.

That's odd, now that I think about it. Given how oddly indistinguishability messes with our everyday perception of how both reality and logic, I suspect that the pop philosophy level has, for the most part, never dug far enough into the math to realize how badly it undermines classical assumptions about uniqueness. You cannot start philosophizing about such an idea until you understand how devastating it is to classical logic.

Cheers,
Terry

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