On the Illusion of Smoothness in Hilbert Space

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2022-05-25.16:20 EDT Wed
https://www.patreon.com/posts/closest-we-have-65854719?cid=84524445

"… higher information density of Hilbert space …"

Sabine, what kind of nonsense are you letting this Bollinger fellow spout THIS time?!? ... oh wait... that's me... :)

My faux pax on this one is an excellent example of how casual, poorly examined biases can impede analysis. Hilbert spaces are infinitely "denser" because they have infinitely more _dimensions_ than 3D or other finite spaces. However, in terms of _data_ that must reside within those infinite dimensions, Hilbert spaces are infinitely dark. As you increase the number of dimensions in a Hilbert space, its information density asymptotically approaches zero as the number of orthogonal states (dimensions) approaches infinity.

For example, it's easy to find one black marble in a row of white marbles, more challenging to find it in a tray of white marbles, harder still to find it in a 3D cube of white marbles... and infinitely difficult to find it in an infinite-dimensional hypercube of white marbles, since at that point the ratio of black to white marbles becomes 1 to infinity.

For any quantum physics model in infinite-dimensional Hilbert space, the issue thus is not that the model has too much specificity or too high of an information density but how to extract _any_ information out of its infinite darkness (that is, infinite volume).

That's where the distribution function concept is helpful [1]. This mathematical technique blurs the data search to include ranges of similar axes (states). However, if you think about that carefully, it's the same as saying the energy scale of the experiment determines the finite number of dimensions needed. The Hilbert formalism becomes an algorithmic pathway for creating more states as needed, such as modeling linear-collider electron-quark collisions.

There's a catch even with using distribution functions in Hilbert spaces. Distribution functions, including their famous limit, the Dirac function, are a delightfully obscure and oblique way of introducing the real-world phenomenon of quantum wave collapse into mathematical models. Distribution functions just let you do this without actually _saying_ the unsightly phrase "wave collapse" out loud. Just as a real-world telescope focused on an Einstein ring can pluck one tiny photon out of a photon wave function that arguably occupies a non-trivial fraction of the entire universe's volume, a distribution function model can pluck that same photon out of the infinite darkness of a sizeable fraction of the volume of the entire universe.

Distribution functions are also popular because they are infinitely differentiable, which make at least local bits of the universe look smooth and shiny at all imaginable scales. Alas, in the end, all distribution functions — including their famous limit, the Dirac function — achieve this infinite smoothness via piecemeal constructions that bump one-over-infinity discontinuities against bits of genuinely smooth finite functions. Such boundaries always involve _computational_ infinities, even if they look smooth to the human eye. This piecemeal approach to creating smooth distribution functions developed over decades of painful and often paradoxical development, and it helps perpetuate the comforting illusion that everything in physics is infinitely smooth, even though no such smoothness ever exists experimentally

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[1] E. P. Xing, Hilbert Space Embeddings of Distributions. Lecture 22 of CMU Course 10-708 Probabilistic Graphical Models (2014).
https://www.cs.cmu.edu/~epxing/Class/10708-14/scribe_notes/scribe_note_lecture22.pdf

Page 1, para 3,4: "Higher order moments bring increasingly greater resolution power for characterizing arbitrary distributions, which leads to the intuition that an infinite dimensional vector consisting of moments will absolutely capture any distribution. This is, of course, practically infeasible, since storing or manipulating a vector of infinite dimensions is impossible. It however motivates the use of Hilbert Space embeddings, and the kernel trick to solve this infinite dimensional representation scenario."

2022-05-25.16:20 EDT Wed
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